

Exam 2 --- 2003 --- SOLUTIONS

I. Multiple choice

1.e., 2.d., 3.a., 4.c., 5.c.

II. a. $k^2 = 2mE/\hbar^2$, $k=8.0 \times 10^9 \text{ m}^{-1}$ (\hbar is meant to be h-bar)
 $\lambda=7.9 \times 10^{-10} \text{ m}$.

b. $\alpha = \text{Square root}(2m|V-E|)/\hbar = 1.6 \times 10^9 \text{ m}^{-1}$.

c. Yes, there is reflection. The stationary solution that is written for the wave on the left must be a superposition of right-going and left-going waves. The solution for $x > 0$ shows a diminishing probability density, but the probability must sum to 1 everywhere, so part of the probability density on the left must come from the reflected wave. Other points of view involving the probability of finding the electron in the forbidden region vs. outside were sensible too.

III. a. Take the 2nd derivative of the wavefunction ψ with respect to x and obtain $-k_1^2 \psi$. The 2nd derivative with respect to y gives $-k_2^2 \psi$. So putting these together gives $-(\hbar^2/2m)(k_1^2 + k_2^2)\psi = E \psi$.

The ψ is a common factor, leaving $-(\hbar^2/2m)(k_1^2 + k_2^2) = E$.

b. At $x=L$ or $y=L$ the wavefunction must vanish. So $k_1 L = n_1 \pi$ and $k_2 L = n_2 \pi$. Hence $k_1 = n_1 \pi/L$ and $k_2 = n_2 \pi/L$.

c. $E_{n_1, n_2} = (\hbar^2/8mL^2)(n_1^2 + n_2^2)$

Lowest energy state has n_1 or $n_2 = 1$. Hence $E_{1,1} = (\hbar^2/4mL^2)$

d. 2 electrons can occupy the lowest energy state. The 3rd electron must go to the next highest level (2,1 or 1,2), giving $2 \times (\hbar^2/4mL^2) + 5(\hbar^2/8mL^2) = 9(\hbar^2/8mL^2)$.

e. Bosons can all occupy the same energy state, so $3 E_{1,1} = (3\hbar^2/4mL^2)$.

IV. a. $1s^2 2s^2 2p^1$

b. $-Z^2 E_0/n^2 = 25 \times 13.6/4 \text{ eV} = 85.0 \text{ eV}$

c. $-Z_{\text{screen}}^2 E_0/4 = 8.28$, so $Z_{\text{screen}} = 1.56$

d. $l=1$, so $\mu = \text{square root}(2) \mu_B$

e. $4f \rightarrow 3d \rightarrow 2p$ so $E(4f) - E(3d) = -Z_{\text{screen}}^2 E_0/16 + -Z_{\text{screen}}^2 E_0/9 = 1.61 \text{ eV}$

$E(3d) - E(2p) = -Z_{\text{screen}}^2 E_0/9 + -Z_{\text{screen}}^2 E_0/4 = 4.6 \text{ eV}$

V. a. $p_{E1} = Qr = 1.60 \times 10^{-19} \text{ Coul} \times 2.36 \times 10^{-10} \text{ m} = 3.78 \times 10^{-29} \text{ Coul m}$

b. measured dipole / theoretical dipole = $3.00/3.78 = 0.79$ or 79%.