

Exam 2 --- 2004 --- SOLUTIONS

I. Multiple choice

1.b., 2.d., 3.c., 4.a., 5.c.

II. a. $k_1^2 = 2mE/\hbar^2$, $k_1 = 3.8 \times 10^{11} \text{ m}^{-1}$ (\hbar is meant to be \hbar)

$\lambda = 1.7 \times 10^{-11} \text{ m}$.

b. Propagates, since $E > V = 2.5 \text{ eV}$. So $\hbar^2 k_2^2 / 2m + 2.5 \text{ eV} = 3.0 \text{ eV}$ or $\hbar^2 k_2^2 / 2m = 0.5 \text{ eV}$.
Then $k_2 = 1.55 \times 10^{11} \text{ m}^{-1}$.

III. a. Take the 2nd derivative of the wavefunction ψ with respect to x and obtain $-k^2\psi$. Then the left hand side of the equation is $-(\hbar^2/2m)(-k^2)\psi$ and the right hand side is just $E\psi$; the ψ cancels out showing that this is a solution, providing the constant factors are equal.

b. $(\hbar^2/2m)k^2 = E$

c. At $x=0$ and L the wavefunction must vanish. So $kL = n\pi$. Hence $k = n\pi/L$.

d. $E_n = (\hbar^2/8mL^2)(n^2)$

e. 2 electrons can occupy the lowest energy state, $n=1$. The 3rd and 4th electron must go to the next highest level, $n=2$, the 5th and 6th to $n=3$, and so on, until all N electrons are accounted for. So the last pair will be in the $n=N/2$ level.

The last electron will thus have energy $E = (N/2)^2 (\hbar^2/8mL^2) = (\hbar^2/32m)(N/L)^2$.

f. Bosons can all occupy the same energy state, so $n=1$ and $E = (\hbar^2/8mL^2)$.

IV. a. $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

Note highest is in $4s$ not $3d$.

b. $-Z^2 E_0 / n^2 = -19^2 \times 13.6 / 4^2 \text{ eV} = 307 \text{ eV}$

c. $-Z_{\text{screen}}^2 E_0 / 16 = 4.34 \text{ eV}$, so $Z_{\text{screen}} = 2.26$

d. Must add 4.34 eV to ionize K. 3.6 eV is released when Cl absorbs electron.

So net of 0.74 eV must be added to pair of atoms.

V. a. $1s$ state is allowed (with opposite m_s to other electron).

$E_{1s} = -Z_{\text{screen}}^2 E_0 / 1 = -24.8 \text{ eV}$.

b. $3d \rightarrow 2p \rightarrow 1s$, each step must have $|\Delta l| = 1$.

c. $E(3d) - E(2p) = -Z_{\text{screen}}^2 E_0 / 9 + Z_{\text{screen}}^2 E_0 / 4 = 3.4 \text{ eV}$

$E(2p) - E(1s) = -Z_{\text{screen}}^2 E_0 / 4 + Z_{\text{screen}}^2 E_0 / 1 = 18.6 \text{ eV}$

Then $\lambda_{3,2} = hc / (3.4 \text{ eV}) = 3.6 \times 10^{-7} \text{ m}$

and $\lambda_{2,1} = hc / (18.6 \text{ eV}) = 6.7 \times 10^{-8} \text{ m}$.