PHYSICS 13 HOUR EXAM 1 FALL 2004

NAME Solutions

This is a closed book, closed notes exam. You may use calculators.

Make sure you show all your work! You will get partial credit for correct intermediate steps.

Useful Formulae and Data

Relativistic Energy = $m_0 c^2/\sqrt{(1-u^2/c^2)}$, where m_0 is the rest mass;

$m_0 c^2$=Rest Energy Relativistic momentum = $m_0 u/\sqrt{(1-u^2/c^2)}$

Lorentz Transformation frame S′ moves in the +x direction with V as seen in S:

$x=\gamma(x'+Vt')$, $y=y'$, $z=z'$, $t=\gamma(t'+Vx'/c^2)$ where $\gamma=1/\sqrt{(1-V^2/c^2)}$

Lorentz contraction: $L=L_0/\gamma$ Time dilation: $t=\gamma t_0$

Addition of velocities for an object moving at u'_x and u'_y in S′:

$u_x=(u'_x+V)/(1+Vu'_x/c^2)$, $u_y=u'_y/(1+Vu'_x/c^2)$

Doppler Effect: $f' = f\,[\sqrt{(1+v/c)}\,/\,\sqrt{(1-v/c)}]$ f = frequency (or ν , Greek nu)

Compton scattering: $\lambda' - \lambda = (h/m_e c)(1-\cos\theta)$; $(h/m_e c)=2.43\times10^{-12}$m

Quantum: E=hf Photoelectric effect: $hf = mv_{max}^2/2 + \Phi$(work function)

de Broglie: $\lambda=h/p$ General waves: $\lambda f=v_{phase}$ Light: $\lambda f=c$

Bohr atom (single electron): $E_n=-Z^2E_0/n^2$, with $E_0=13.61\text{ eV}=k^2e^4m/2\hbar^2$

transitions: $hf_{nm} = E_n - E_m$

Heisenberg's Uncertainty: $\Delta x\Delta p \geq \hbar/2$

Constants: Velocity of light: c = 3.00×10^8 meter/sec

Planck's constant: h = 6.63×10^{-34} J sec= 4.14×10^{-15} eV sec

$\hbar = h/2\pi = 1.05\times10^{-34}$ J sec=6.58×10^{-16} eV sec $\hbar c=1.97\times10^{-7}$ eV m

hc = 1.24×10^{-6} eV m Rest energy of electron = $m_e c^2$ = 0.511 MeV

Units: Energy: 1 J(oule) = 1 Kg m²/sec² ; 1 eV(electron Volt) = 1.6×10^{-19} J;

1 MeV = 10^6 eV = 1.78×10^{-30} Kgc² & 1 MeV/c² = 1.78×10^{-30} Kg

Grades:	points	/possible points
I		/50
II		/40
III		/50
IV		/60
Total:		/200

Avg ≃ 150

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I. Multiple choice -- circle the one best answer.

1) The Heisenberg Uncertainty Principle
   a. verifies the wave nature of EM radiation.
   b. puts a lower limit on momentum uncertainty for given position uncertainty.
   c. puts an upper limit on momentum uncertainty for given position uncertainty.
   d. verifies the particle nature of electrons.
   e. puts an upper limit on position uncertainty for given momentum uncertainty.

2) The "twin paradox" is not a paradox. It is resolved because
   a. the travelling twin does not actually age slower.
   b. the stationary twin is not always in an inertial frame of reference.
   c. the travelling twin is not always in an inertial frame of reference.
   d. the slowing down of moving clocks is an illusion.
   e. the stationary twin is actually younger when the twins are reunited.

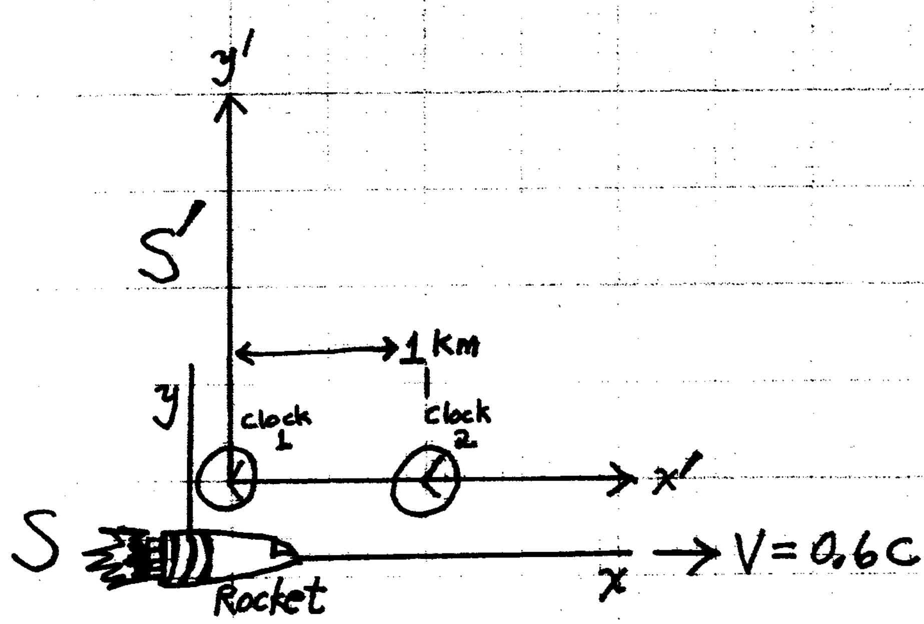
3) Which of the following is not true of Blackbody Radiation?
   a. The radiated power decreases as the wavelengths become very short for fixed T.
   b. The maximum power occurs at shorter wavelengths for rising T.
   c. The power spectrum demonstrates the existence of quanta of energy with $E=hf$.
   d. The power spectrum demonstrates the ultraviolet catastrophe.
   e. The Classical (Stefan) law that radiated power $\propto T^4$ applies.

4) A satellite is in a distant stable orbit around the earth. In accord with the Equivalence Principle, a set of simple, small, brief experiments conducted by astronauts inside the satellite would be consistent with the conclusion that
   a. the inertial mass of objects increases in time.
   b. light travels along hyperbolic paths.
   c. light speed increases when traveling perpendicular to the motion.
   d. this is an inertial frame of reference, far from external gravitational forces.
   e. measuring sticks in the cabin appear shorter in the direction of motion compared to the direction perpendicular to the motion.

5) A neutron has rest energy 940 MeV and average lifetime at rest of 15 minutes. Suppose a beam of neutrons is accelerated to an energy of 9,400 MeV. Which statement is true of a neutron in the beam?

a) Its speed is a small fraction of c.
b) Its momentum is m(proton) c.
c) Its rest mass increases.
d) Its lifetime in the lab is 1.5 minutes.
e) Its lifetime in the lab is 150 minutes.

II. Suppose there are two space stations at rest relative to each other and separated by a distance of 1 Km. Call their "rest frame" **S´**. Their clocks are synchronized and at time **t´=0** both clock faces light up.
A rocket is traveling at 0.6c along the line connecting the two space stations. At time t=0 (in the rocket's frame of reference **S**) the rocket reaches the first station just as its clock lights up.



a. According to the rocket, how far away is the other space station?

$\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} = \frac{5}{4}$ $\quad L = L'/\gamma = \frac{4}{5}L' = 0{,}8 \text{ Km}$

II.b. According to the rocket (i.e. in the rocket's frame of reference), at what time does the second clock face light up? Does this second event occur before, at the same time, or after the first event?

$$t = \gamma\left(t' - \frac{v}{c^2}x'\right) = -\gamma \frac{v}{c}\frac{x'}{c} = -\frac{5}{4} \times \frac{3}{5} \times \frac{10^3 m}{3\times 10^8 m/s}$$

$$= -\underline{2.5 \times 10^{-6} \sec}$$

before 1st event (in Rocket frame)

c. Suppose a second rocket is going at 0.6c in the opposite direction, and coincides with the first rocket at the second rocket's time 0 (i.e. all the three reference frames' clocks read zero when their origins coincide). At what time does the second event occur according to this rocket's time measurement? Is it before, simultaneous with, or after the first event?

$$t'' = \gamma\left(t' + \frac{v}{c^2}x'\right) = \underline{+2.5 \times 10^{-6} \sec}$$

after 1st event (in 2nd Rocket frame)

$(v'' = -v = -0.6c)$

d. According to the first rocket, what is the velocity of the second rocket?

$$v_2 = \frac{v'' + v}{1 + \frac{v v''}{c^2}} = \frac{1.2c}{1 + (.6)^2} = \frac{1.2}{1.36}c = \underline{0.88c}$$

III. Consider the Bohr model of the singly ionized **Helium** atom (Helium has **Z=2**).

a. What are the <u>three</u> lowest allowed energies of the electron?
Call these E_1, E_2, E_3. Use eV units.

$$Z=2 \quad E_n = -\frac{Z^2 E_0}{n^2} = -\frac{4 \times 13.6\,\text{eV}}{n^2}$$

$E_3 = -6.0\,\text{eV}$ ✓

$E_2 = -13.6\,\text{eV}$ ✓

$E_1 = -54.4\,\text{eV}$ ✓

b. What are the <u>different energies</u> of photons that can be emitted when the electron undergoes transitions starting from the third level E_3 to lower energy levels?

$3 \to 2$: $E_3 - E_2 = 13.6 - 6.0 = 7.6\,\text{eV}$ ✓

$2 \to 1$: $E_2 - E_1 = 54.4 - 13.6 = 40.8\,\text{eV}$ ✓

$3 \to 1$: $E_3 - E_1 = 54.4 - 6.0 = 48.4\,\text{eV}$ ✓

c. Which of the preceding <u>transitions</u> results in the <u>highest frequency</u> photon? What is that <u>frequency</u>? $\underline{3 \to 1}$ releases largest E photon

$$f = \frac{E}{h} = \frac{48.4\,\text{eV}}{4.14 \times 10^{-15}\,\text{eV}\cdot\text{sec}} = \underline{1.17 \times 10^{16}\ \text{sec}^{-1}}$$

d. When this highest frequency light is incident on a certain metal, the photoelectric effect occurs. The <u>work function</u> of the metal is 5.0 eV. What is the <u>stopping potential</u> for this case?

$$hf = KE_{max} + \Phi$$

$$V_{stop} = KE_{max} = hf - \Phi = (48.4 - 5.0)\,\text{eV}$$

$$= \underline{\underline{43.4\,\text{eV}}}$$

III.e. Assume that same frequency light is emitted from a distant star, traveling away from earth at 1/2 the speed of light. What frequency would be seen on earth?

$$f' = f\sqrt{\frac{1-v/c}{1+v/c}} = f\sqrt{\frac{1/2}{3/2}} = \frac{f}{\sqrt{3}} = 6.75 \times 10^{15}\ sec^{-1}$$

IV. An electron, not localized in space, is described by a wavelike solution to the Schrödinger wave equation. Suppose a free electron, with (non-relativistic) kinetic energy of 1000 eV, has a wavefunction given by

$\Psi(x,t) = A \sin(kx-\omega t)$.

where $k=2\pi/\lambda$ is the wavenumber, $\omega=2\pi f$ is the angular frequency and A is an unspecified constant.

a. What is the relation between the wavenumber k and the kinetic energy?

$$KE = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \quad \left(\text{because } p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k\right)$$

$$k = \frac{\sqrt{2m\,KE}}{\hbar}$$

b. Find the numerical value of k for this 1000 eV electron wave.

$$1000\,eV = \frac{\hbar^2}{2m}k^2 = \frac{(\hbar c)^2}{2mc^2}k^2 = \frac{(1.97\times10^{-7}\,eV\cdot m)^2\,k^2}{2(0.511\times10^{6}\,eV)}$$

$$= (3.797\times10^{-20}\,eV\,m^2)\,k^2$$

$$k = \left(\frac{10^3\ eV}{3.797\times10^{-20}\,eV\,m^2}\right)^{1/2} = 1.62\times10^{11}\ m^{-1}$$

c. Find the numerical value of ω for this electron wave.

$$hf = \hbar\omega = 10^3\,eV \rightarrow \omega = \frac{10^3\,eV}{6.58\times10^{-16}\,eV\cdot sec}$$

$$= 1.52\times10^{18}\ sec^{-1}$$

IV.d. What is the value of the velocity of this electron wave?

$$v_{wave} = \lambda f = \frac{2\pi f}{2\pi/\lambda} = \frac{\omega}{k}$$

$$= \frac{1.52\times10^{18}\ \mathrm{sec}^{-1}}{1.62\times10^{11}\ \mathrm{m}^{-1}} = 0.938\times10^{7}\ \mathrm{m/sec}$$

$$= 9.38\times10^{6}\ \mathrm{m/sec}$$

e. If the kinetic energy is that of a classical particle, what value of velocity would you get?

$$KE = \frac{p^2}{2m} = \frac{1}{2} m v_{cl}^2$$

$$v_{cl} = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\times KE}{mc^2}}\ c = \sqrt{\frac{2\times10^3\,\mathrm{eV}}{0.511\times10^6\,\mathrm{eV}}}\ c$$

$$= 0.0626c = 1.88\times10^{7}\ \mathrm{m/sec}$$

$$\text{or } \underline{2\times v_{wave}}$$

f. At time t=0, what is the probability of finding this electron within an interval of ± dx/2 from the position x= 10^{-11}m? Express your result in terms of A and dx.

$$|\psi(x,0)|^2 dx = |\psi(10^{-11}\mathrm{m},0)|^2 dx$$

$$= A^2 \sin^2(k\times10^{-11}\mathrm{m},0)\ dx = A^2 \sin^2\left(\frac{\pi}{2}\right) dx$$

$$= \underline{A^2 dx}$$