PHYSICS 13 FINAL EXAM FALL 2003

NAME____

Useful data and equations: Schrödinger equation: (η=h/2π)

$$-\frac{\acute{h}^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

For two interacting bodies, the m is replaced by reduced mass μ and x is the separation.

Quantum mechanical angular momentum $|\mathbf{L}| = [l(l+1)]^{1/2}\eta$ and $L_z = m_l\eta$ Note the **h-bar**

or more generally $|\mathbf{J}| = [j(j+1)]^{1/2}\eta$ and $J_z = m_j\,\eta$ with m_j = j, j-1, ..., -j

Addition of orbital (l) and spin (1/2) angular momenta yields j=l-1/2, l +1/2.

Addition of $\mathbf{J}_1$ and $\mathbf{J}_2$ yields sum $\mathbf{J}$ with quantum number $j = |j_1-j_2|,\ |j_1-j_2|+1, ..., j_1+j_2-1, j_1+j_2$

Atomic electron spectroscopic notation: $n\,l^{\text{1 or 2 or...}}$, where n is principal quantum number, l is orbital angular momentum quantum number, l=0,1,2,...,n-1, with notation l=0->s, 1->p, 2->d, 3->f,...

Avogadro's Number: $N_A = 6.0 \times 10^{23}$ number of atoms, molecules or nuclei in a MOLE (or gram molecular wt)

Non-relativistic Kinetic Energy $=mv^2/2$ Magnetic dipole interaction energy = $\mu_z B_z$

Lorentz Transformation frame S´ moves in the +x direction with V as seen in S:

$$x=\gamma(x'+Vt'),\ y=y',\ z=z',\ t=\gamma(t'+Vx'/c^2) \quad \text{where} \quad \gamma=1/\sqrt{(1-v^2/c^2)}$$

Doppler effect $\lambda' = \lambda\sqrt{(1+v/c)}/\sqrt{(1-v/c)}$ Time dilation $t' = \gamma t$

Relativistic Energy $E= m_0c^2/\sqrt{(1-v^2/c^2)}$; Rest energy = m_0c^2 ; Relativistic Kinetic Energy = E - m_0c^2

Photon: E=hf ; c=λf ; k=2π/λ ; ω =2πf Electric dipole moment p_{E1}= Q r (Q=charge, r=separation)

Average Thermal Energy at Temperature T(Kelvin) = k(Boltzmann)T/2 per degree of freedom

Decay N(t) = N(0)exp(-λt), λ =1/(mean lifetime) = $\ln 2/T_{1/2}$ where $T_{1/2}$ is the half-life

Nuclear Binding Energy $E_b = (Zm(p) + Nm(n) - M_A)\,c^2$ Coulomb Potential $k_{EM}Q_1Q_2/r$

Newton Law of Gravitation : potential energy = $-Gm_1m_2/r$

Constants: Velocity of light: c = 3.00x 10^8 meter/sec electron charge: e=1.60x10^{-19} Coul

Newton gravitational constant G = 6.67x10^{-11}J*m/Kg^2

k(Boltzmann) = 1.38x10^{-23}J/°K = 8.62x10^{-5}eV/°K

Planck's constant: h = 6.63 x 10^{-34} J sec= 4.14 x 10^{-15} eV sec

η = h/2π = 1.05 x 10^{-34} J sec = 6.58 x 10^{-16} eV sec $k_{EM} = 1/4\pi\varepsilon_0$ = 8.99x10^9 Nm^2/Coul^2

hc = 1.24 x 10^{-6} eV m Rest energy of electron = m_ec^2 = 511 keV = 0.511 MeV m_e=9.1x10^{-31}Kg

Unified mass units 1 u = 1.66x10^{-27} Kg (1u)c^2 = 931.5 MeV

Units: Energy: 1 J(oule) = 1 Kg m^2/sec^2 ; 1 eV(electron Volt) = 1.6x10^{-19} J

CHOOSE 2 OUT OF LAST 3 PROBLEMS (V, VI, VII) i.e drop one **EXTRA CREDIT Prob.VIII

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This is a closed book, closed notes exam. You may use calculators.

Make sure you show all your work! You will get partial credit for correct intermediate steps.

Grades: points	/possible points	
I	/100	
II	/50	
III	/40	
IV	/40	
V	/30*	**CHOOSE 2 OUT OF LAST 3 (V,VI,VII) i.e drop one**
VI	/30*	
VII	/30*	
Total:	/290	

****EXTRA CREDIT PROBLEM VIII for 20 pts** **

I. Multiple choice -- circle the one best answer.

1) The “Curve of Binding Energy” is a graph of the magnitude of the binding energy per nucleon (BE/A) of the neutrons and protons plotted against mass number A. The lightest nuclei have smaller BE/A than

a. deuterons, which makes fission possible.
b. deuterons, which makes fusion possible.
c. medium mass number nuclei, which makes fission possible.
d. medium mass number nuclei, which makes fusion possible.
e. electrons in atoms.

2) The binding energy of the neutron and proton in a Uranium nucleus is larger than the typical atomic or molecular binding energy by a factor of

a. 32 b. roughly $6x10^{23}$ c. roughly 10^{6} d. roughly 100 e. roughly $1/6x10^{23}$

3) In Grand Unified Theories of particles the proton is unstable. In one such theory the mean lifetime is 10^{32} years (age of universe is ~ 10^{10} years). That being the case, which statement is most true?

a. Protons will not be seen to decay in our lifetime.
b. A single proton is very likely to decay in 10 years.
c. 1 proton in a sample of e^{32} will probably decay in a year.
d. All protons in a sample of e^{32} protons will probably decay in the mean lifetime.
e. No protons have decayed yet in the universe.

4) The **nucleons** are held together in a **nucleus** by

a. gravitational interaction
b. electromagnetic interaction
c. electroweak interaction
d. weak interaction
e. strong interaction

5) Consider the element ${}_{10}Ne$. Its ground state is

a. $1s^2 1p^6 2s^2$ b. $1s^2 2s^2 3s^2 3p^4$ c. $1s^2 2s^2 2p^6$ d. $1s^2 2s^2 2p^4 2d^2$ e. $1s^2 3d^8$

6) A free particle of kinetic energy (KE) 3 eV, traveling in a straight line, approaches a constant higher potential of +2.5 eV. The particle will

a. have a 100% probability to continue along at KE =0.5 eV.
b. be reflected back at the boundary with 100% probability.
c. have a 100% probability to decay exponentially in the barrier.
d. lose energy in the 2.5 eV region until it stops.
e. be partially reflected with KE=3 eV and partially transmitted with KE=0.5 eV.

7) A particular crystalline solid has an **energy gap** of 5.0 eV between the valence band and the nearest conduction band. At T≅□0K the valence band is filled and the conduction band is empty. At room temperature (T≅□300K, kT≅□0.03 eV)

a. many electrons will be in the conduction band, making this a semiconductor.
b. many electrons will be in the conduction band, making this an insulator.
c. very few electrons will be in the conduction band, making this a semiconductor.
d. few electrons will be in the conduction band, making this an insulator.
e. the valence and conduction bands will overlap, making this like a metal.

8) Quarks are elementary particles that

a. only interact gravitationally
b. only interact electromagnetically
c. are constituents of electrons
d. have the charge of the electron and positron
e. are constituents of protons and neutrons

9) The current theory of elementary particles, the Standard Model, includes

a. 3 colored quarks and 8 colored leptons.
b. 3 colored quarks and 8 colored gluons.
c. gravitational interactions of the quarks with the leptons.
d. the unification of all the forces of nature.
e. only the weak interaction.

10) The Big Bang theory of the origin of the universe is in agreement with many observations and predictions. Which of the following is **not** a true observation or prediction that confirms the theory?

a. The most distant observed galaxies are in much earlier stages of formation than the Milky Way.
b. The universe is filled with blackbody radiation at temperature 2.7K.
c. The universe is nearly flat.
d. Galaxies recede from us with velocities proportional to their distance.
e. Einstein showed that a stationary universe is the only solution to his field equations of General Relativity.

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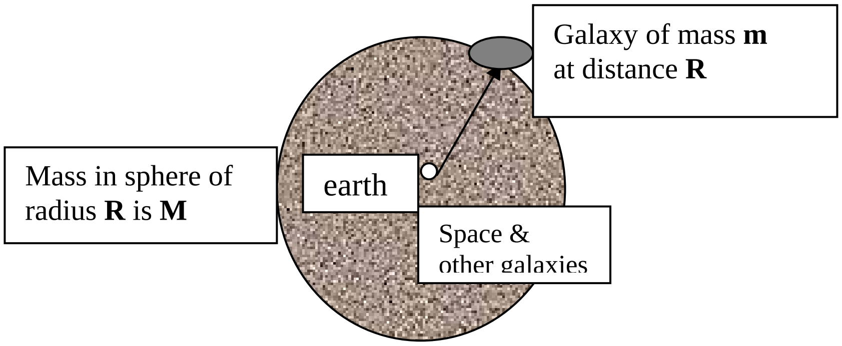
II. Hubble's Law relates the receding velocity of a galaxy to its distance from Earth via **v =(dR/dt)= HR** where the Hubble constant H = (23Km/sec)/(10^6 light years) at the current time.

a. What is the speed of the galaxy G40 that is 5 billion light years away?

b. What will be the wavelength measured on Earth of the 590.0 nm Sodium line emitted from stars in G40?

c. Suppose the mass of a nearer galaxy at distance **R** is **m** and the total mass of all matter within a sphere of radius **R** (centered on the Earth) is **M**. In the ***non-relativistic*** Newtonian theory of gravity, the gravitational potential energy U(R) of the galaxy is (- GmM/R) . Using Hubble's Law, express the galaxy's non-relativistic kinetic energy K(R) in terms of its distance R.



d. For a flat universe the kinetic energy of the distant galaxy is exactly balanced by the potential energy. Use that condition to get an expression for the **critical density** of matter in the universe, ρ_{critical}. Express this in terms of **H** and **G**, the Newtonian gravitational constant.

e. Evaluate that critical density in units of protons per cubic meter. The mass of a proton is 938 MeV/c^2 or $1.67\text{x}10^{-27}$Kg.

III. The quantum mechanical explanation of many properties of metallic filaments can be modeled by a system of electrons confined to a one dimensional tube or wire of length L. Treat the electrons as non-interacting fermions.

a. $\psi(x) = (2/L)^{1/2} \sin(kx)$ is a normalized solution to the 1-dimensional stationary Schrödinger equation for a <u>free electron</u> with kinetic energy $\eta^2(k^2)/2m$.
Under what condition on k will this $\psi(x)$ be a wavefunction for one electron in the tube of length L? (You will need to introduce a quantum number for this system; call it n.)

III.b. What are the allowed energies E(n) for a single electron in terms of n?

c. Suppose electrons are added to the tube, one at a time, by putting each electron in the lowest allowed energy state. When there are N electrons in the box, what is the <u>lowest possible energy for the Nth electron</u> (assume N is a very large number and remember that electrons have spin 1/2)? Call this energy E_F^{linear}. Express your answer in terms of h, m (mass of the electron), L and N.

d. Aluminum has a free electron number density of $1.81 \times 10^{23}/\text{cm}^3$. Taking the 1/3 power of this gives 5.66×10^7/m (or 56.6 electrons per micron) as the *linear* number density. What is the Fermi Energy E_F^{linear} of an Al filament in this 1-dimensional model? Use units of eV.

IV. The fission process $n + {}^{235}_{92}U \rightarrow {}^{90}_{37}Rb + {}^{144}_{?}Cs + ?n$
releases energy that you will calculate.

a. How <u>many neutrons</u> must be released? What is Z, the atomic number, for Cs (cesium)?

b. The masses of the nuclei are given below. What is the <u>amount of mass</u> that disappears (i.e. gets converted to other forms of energy) in the reaction? (Be sure to account for the neutrons.)

$M(n) = 0.0167 \times 10^{-25}$ Kg
$M(^{235}U) = 3.9184 \times 10^{-25}$ Kg
$M(^{90}Rb) = 1.4925 \times 10^{-25}$ Kg
$M(^{144}Cs) = 2.3892 \times 10^{-25}$ Kg

c. The mass that disappears is converted to how much <u>energy</u>? Express your answer in <u>Joules</u> or <u>MeV units</u>.

d. How much <u>energy</u> would be produced if about 1/2 Kg of the Uranium undergoes fission, i.e. every nucleus in 1/2 Kg undergoes fission? You'll need to find the number of U nuclei in that mass first. Express your answer in Kilotons.
(1 Kiloton = 4.2×10^{12} J = 2.6×10^{25} MeV. 1 Kiloton is the equivalent of one thousand tons of TNT explosives.)

V. Consider the potassium flouride KF molecule. Suppose it is primarily ionic.
a. The ionization energy (energy to remove one electron) for K is 4.34 eV. The electron affinity (energy released when one electron is added) for F is 3.45 eV. When the neutral atoms K and F are separated to infinity (or a very large distance R), how much energy does it take to create the ions K^+ and F^- ?

V.b. Calculate the Coulomb potential energy for K^+F^- for which the equilibrium separation is 0.217 nm. Assume the two ions are point charges.

c. The measured dissociation energy (to pull apart the molecule into neutral K and F atoms) is 5.07 eV. Given your answers to parts (a) and (b), how much energy is associated with repulsion of the ions at equilibrium separation?

VI. Top quarks are the most massive elementary particles with mass = 175 GeV/c^2. Suppose many top quarks are produced in an accelerator laboratory with an average total energy of 245 GeV by special reactions. Top quarks decay with a **half life** of about $7.0x10^{-26}$ sec.

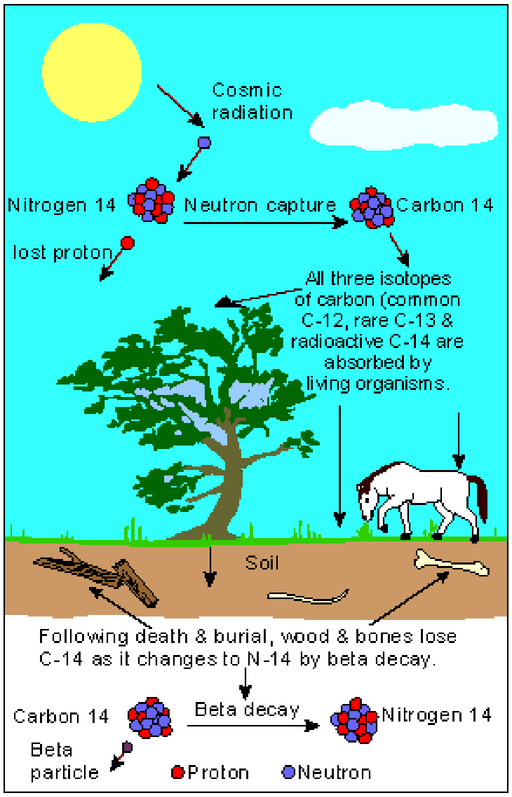
a. What is the average speed of a top quark in the accelerator laboratory? (Hint: Use the relativistic energy vs. the rest energy.)

b. What is the **half life** measured in the accelerator laboratory?

c. Those top quarks travel in the lab and decay as they go. Suppose 1/2 of them have not decayed after traveling a certain distance. What is that distance?

VII. Carbon 14 (^{14}C) is radioactive and undergoes beta decay. It has a half life of 5730 Years.

a. What is its **mean lifetime (1/λ)**?



VII.b. A piece of wood from a recently cut tree shows 12.4 ^{14}C decays per minute. A sample of the same size from a tree cut thousands of years ago shows 3.5 decays per minute. What is the age of the sample?

b. How many atoms of ^{14}C are in the sample from the old tree?

************ EXTRA CREDIT** Try this ONLY if you have finished the others.********

VIII. Consider a diatomic molecule, like NaCl.

a. For small oscillations x, about the equilibrium separation, the Schrödinger equation applies, with the mass being the reduced mass μ and the potential being the simple harmonic oscillator potential $V(x) = (1/2)\ \mu\ \omega_0^2 x^2$. For the classical oscillator, the resulting motion has angular frequency ω_0. Show by substitution that $\psi_0(x) = \exp[-(\mu\ \omega_0 x^2)/2\eta]$ can be a solution to the appropriate Schrödinger equation.

b. From the substitution in (a), find the corresponding energy E_0 for this vibrational state in terms of ω_0 and η?