| Grades: | points/possible points |
|---|---|
| I | /50 |
| II | /50 |
| III | /40 |
| IV | /60 |
| Total: | /200 |

This is a closed book, closed notes exam. You may use calculators. **Make sure you show all your work! You will get partial credit for correct intermediate steps.**

Useful Formulae and Data

Relativistic Energy = $m_0 c^2/\sqrt{(1-u^2/c^2)}$, where $m_0$ is the rest mass;

$m_0 c^2$=Rest Energy

Lorentz Transformation frame S´ moves in the +x direction with V as seen in S: $x=\gamma(x'+Vt')$, $y=y'$, $z=z'$, $t=\gamma(t'+Vx'/c^2)$ where $\gamma=1/\sqrt{(1-V^2/c^2)}$

Addition of velocities for an object moving at $u'_x$ and $u'_y$ in S´:

$u_x=(u'_x+V)/(1+Vu'_x/c^2)$, $u_y=u'_y/(1+Vu'_x/c^2)$

Doppler Effect: $f' = f\,[\sqrt{(1+v/c)} / \sqrt{(1-v/c)}]$ f = frequency

Compton scattering: $\lambda' - \lambda = (h/m_e c)(1-\cos\theta)$; $(h/m_e c)=2.43\times10^{-12}$m

Quantum: E=hf Photoelectric effect: $hf = mv_{max}^2/2 + \Phi$(work function)

de Broglie: $\lambda=h/p$ General waves: $\lambda f=v_{phase}$ Light: $\lambda f=c$

Bohr atom (single electron): $E_n=-Z^2E_0/n^2$, with $E_0=13.61$ eV$=k^2e^4m/2\eta^2$

transitions: $hf_{nm} = E_n - E_m$

Heisenberg's Uncertainty: $\Delta x\,\Delta p \geq \eta/2$

Constants: Velocity of light: c = $3.00\times 10^8$ meter/sec

Planck's constant: h = $6.63 \times 10^{-34}$ J sec= $4.14 \times 10^{-15}$ eV sec

$\eta = h/2\pi = 1.05 \times 10^{-34}$ J sec = $6.58 \times 10^{-16}$ eV sec

hc = $1.24 \times 10^{-6}$ eV m Rest energy of electron = $m_ec^2$ = 0.511 MeV

Units: Energy: 1 J(oule) = 1 Kg $m^2/sec^2$ ; 1 eV(electron Volt) = $1.6\times10^{-19}$ J;

1 MeV = $10^6$ eV = $1.78\times10^{-30}$ Kg□$c^2$ & 1 MeV/$c^2$ = $1.78\times10^{-30}$ Kg

NAME________________________

I. Multiple choice -- circle the one <u>best</u> answer.

1) In Special Relativity which statement is true?

a. The Laws of Physics require an absolute frame of reference.
b. The Laws of Physics have the same form in all inertial frames.
c. The speed of light depends on the motion of the source.
d. The speed of light depends on the motion of the inertial observer.
e. A massive object can be accelerated exactly to the speed of light.

2) A constant force **F** acts on an object of rest mass $m_0$ for an indefinitely long period of time.

a. The object's inertia or "relativistic mass" increases indefinitely, but its velocity approaches c in magnitude from below.
b. The object's acceleration remains constant.
c. The object's inertia or "relativistic mass" increases indefinitely, and its velocity increases linearly.
d. The object's rest mass increases indefinitely.
e. The object acceleration increases indefinitely.

3) An unpowered spaceship is falling directly toward the earth. In accord with the Equivalence Principle, experiments <u>inside a closed cabin</u> in this free-falling spaceship would indicate that

a. the inertial mass of objects increases in time.
b. light travels along parabolic paths.
c. light undergoes a red shift when travelling from ceiling to floor.
d. this is an inertial frame of reference, far from external gravitational forces.
e. measuring sticks in the cabin appear shorter in the direction of motion compared to the direction perpendicular to the motion.

4) Which statement is <u>not</u> true in quantum theory?

a. A localized electron can be described as a superposition of many travelling sine waves.
b. An electron confined to a one dimensional box (a narrow tube) always has non-zero energy.
c. A photon scattered by an electron can become lower in frequency.
d. A beam of photons directed at a double slit will show diffraction.
e. A beam of electrons directed at a double slit will not show diffraction.

5) The Compton Effect

a. verifies the wave nature of EM Radiation.
b. predicts a lower frequency for back - scattered light.
c. predicts a higher frequency for back - scattered light.
d. shows that there is no ether.
e. depends on the Fourier series for a moving electron.

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II. $K^+$ mesons (positive kaons) decay into other particles, particularly pions. Their proper mean lifetime (i.e. the average time they exist before decaying) is $\tau_0 = 1.24\text{x}10^{-8}$ sec and their rest energy is 494 MeV. Suppose a beam of kaons is **created** in an accelerator laboratory with velocity v=0.9c .

a. What is the energy of these kaons in the laboratory?

b. What will the kaon's mean lifetime be in the laboratory? Is it longer or shorter than $\tau_0$ ?

c. How far will a kaon travel (on the average) in the lab's frame of reference before it decays?

d. Suppose the kaon beam is directed at another kaon beam with the same speed but moving in the opposite direction, so that a head on collision occurs as determined in the laboratory. What is the speed of the second kaon beam as determined in a frame of reference moving along with the first kaon beam?

K beam 1 K beam 2

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$v_1$ = 0.9 c $v_2$ = - 0.9 c

e. What is the <u>total energy</u> of a colliding pair of kaons, one from each beam, as measured in the lab?

III. Consider the Bohr model of the doubly ionized **Lithium** atom (Lithium has **Z=3**).

a. What are the <u>three</u> lowest allowed energies of the electron? Call these $E_1$, $E_2$ , $E_3$. Use eV units.

b. What are the <u>different energies</u> of photons that can be emitted when the electron undergoes transitions starting from the third level $E_3$ to lower energy levels?

c. Which of the preceding <u>transitions</u> results in the <u>longest wavelength</u> photon? What is that longest wavelength?

d. When this longest wavelength light is incident on a certain metal, the photoelectric effect occurs. The work function of the metal is 1.9 eV. What is the stopping potential for this case?

IV. A molecule of mass $2.0 \times 10^{-24}$ Kg is moving along at a speed of **100±10 m/sec**, where the ±**10 m/sec** is the statistical spread or uncertainty in the determination of its **velocity** or **$\Delta$v = 10 m/sec**. The molecule is somewhat localized in space and is described by a **travelling wave packet** solution to the Schrödinger wave equation centered around x=0 at t=0 and moving in the positive x direction.

a. What is the minimum statistical spread or uncertainty **$\Delta$x** in the determination of its position?

b. What is the de Broglie wavelength of this molecule?

c. What is the (non-relativistic) kinetic energy of this molecule?

d. Using part(c), determine the frequency **f** of this travelling wave packet.

e. Does the frequency f in part(d) satisfy the relation $\lambda f = v$ with the given central value of v=100 m/s? Show your calculation.

f. Suppose this electron has a wavefunction given at time t=0 by

$$\Psi(x,0) = A \sin((2\pi/\lambda)\, x)\ .$$

What is the probability density at $x=\lambda/4$?
(Write your answer in terms of A and $\lambda$.)