1. Physics 13: Electrical and Thermal Properties -Metals

Physics 13: Electrical and Thermal Properties - Metals

- Classical free electron theory - Ohm's Law
  - But fails: magnitude of ρ (resistivity) or σ (conductivity)
  - $\rho_{classical}(T) \sim v_{ave}(T) \sim \sqrt{T}$ but $\rho_{msd}(T) \sim T$ & $\rho_{msd} \sim \rho_{class}/10$
    - QM electron gas: v_{ave}~independent of T (for $kT << E_{fermi}$)
    - ρ depends on wave-like e^- s propagating in lattice
  - Why conductors and insulators?
  - Heat conduction and capacity are incorrect
  - Classical e^- gas: $eE-F_{drag}=0$ & $F_{drag} \sim v_{drift}$ /(collision time τ) among ions. So $v_{drift} \sim E\,\tau$. Current $j \sim nev_{drift}$. Ohm's law & resisitivity $\rho \sim v_{drift} \sim 1/\tau$. But $\tau \sim 1/v_{AVE}$ so $\rho \sim v_{AVE}$ & from stat.mech. $v_{AVE} \sim \sqrt{T}$. What is wrong with this picture?

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2. Electrical & Thermal properties - QM

Electrical & Thermal properties - QM

- e^-s obey exclusion principle - occupy states of solid lattice accordingly - even at T=0ºK they have finite E's
- 1-dim Square well model at 0ºK

  $E_1=h^2/8m_eL^2$ & for N e^-s each state ($E_n=n^2E_1$) is occupied by a pair

  Highest E for N/2 or $E_{Fermi}(T=0K)=E_{N/2}=N^2 E_1/4$

$$\text{or } E_{Fermi}(T=0K) = \frac{h^2c^2}{32m_ec^2}\left(\frac{N}{L}\right)^2 = \frac{(1240 \text{ eV}\cdot\text{nm})^2}{32(5.11\times10^5 \text{ eV})}\left(\frac{N}{L}\right)^2$$
$$= 9.40\times10^{-2}\text{eV}\cdot\text{nm}^2\left(\frac{N}{L}\right)^2$$

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3. 1-dim Fermi level example - Cu

1-dim Fermi level example - Cu

Cu has ~ 1 e^- per atom. Density=8.92 g/cm^3 (tables).
Molar mass=63.5 g/mole. So 8.92/63.5=0.140mole/cm^3.
Then N/V=0.140×(6.02×10^{23})=8.43×10^{22} e^-/cm^3.
Taking a cubic lattice to get 1-d
N/L=(8.43×10^{22})$^{1/3}$=4.39 ×10^7/cm=4.39/nm.
So 1-d Fermi level is
$E_{Fermi}=9.4\times10^{-2}\times(N/L)^2=9.4\times10^{-2}\times(4.39)^2=1.8$ eV (?)
This 1-dim model result -> right order of magnitude
Compare that to kT(300K)=2.6×10^{-2} eV.

4. 3-dim Fermi level

3-dim Fermi level

- In 3-d Fermi energy is altered - e^-s now fill a spherical well (real potentials?)

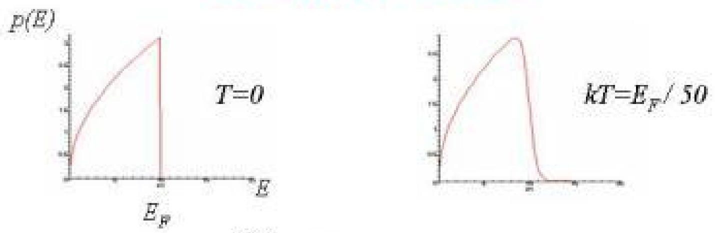
$$E_{Fermi}(T=0K)=\frac{h^2c^2}{8m_ec^2}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Examples:	N/V(cm^3)	E_F(eV)
Al	18.1×10^{22}	11.7
Cu	8.47×10^{22}	7.04
K	1.40×10^{22}	2.11

E_F(0K)>>kT(300K) (or 0.026eV)

5. Density of states

Density of states



At T = 0K $p(E) = \frac{3N}{2} E_F^{-3/2} E^{1/2}$ for $E \le E_F$ and 0 otherwise.

$$\int_0^{E_F} p(E)dE = N \quad \text{and} \quad E_{ave} = \int_0^{E_F} Ep(E)dE / N = \frac{3}{5} E_F$$

At finite T have Fermi-Dirac factor $\frac{1}{e^{\frac{E-E_F}{kT}}+1}$ also

Only small fraction of e's ($\sim kT/2E_F$) above E_F can move to other states in response to Electric field.

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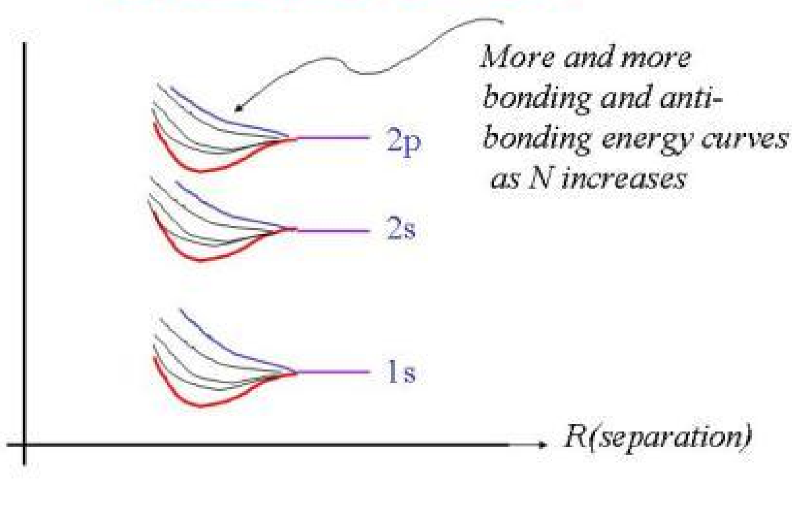
6. Filling energy states for lattice of atoms (beyond square we...

Filling energy states for lattice of atoms (beyond square well model)



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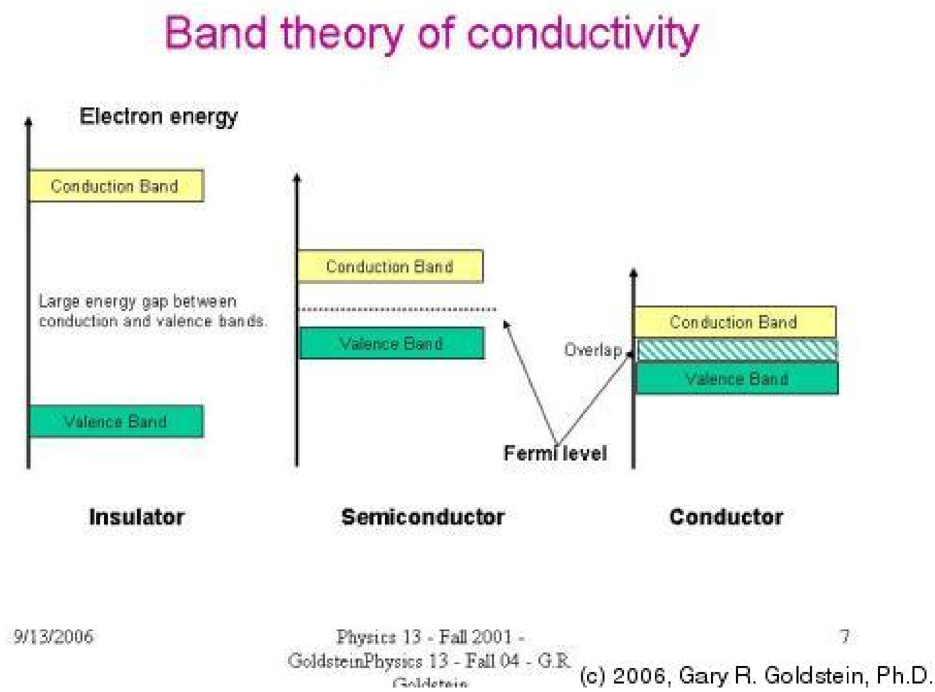
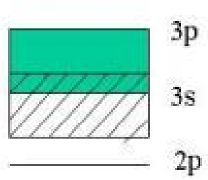
7. Band theory of conductivity



8. Examples - Na, Mg - Conductors

Examples - Na, Mg - Conductors

- Na atom K & L shells filled + $3s^1$ valence *e*'s
- In solid 1/2 of 3s Band is filled
  - 3s electrons are free to move, i.e. jump to empty states $\Rightarrow$ conductor
- Mg: atomic $3s^2$ valence *e*'s
- In solid 3s Band has some overlap with 3p Band $\Rightarrow$ conductor



Unfilled states in 3p Conduction Band
Filled states in 3s Valence Band

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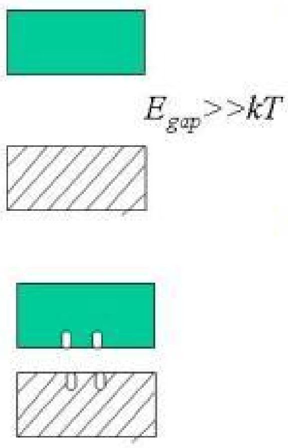
9. Insulators & semiconductors

Insulators & semiconductors



- Conduction band empty
  Valence band full
  Big gap ⇒ Insulator
  (e.g. NaCl, C)
- Small gap ~kT - some e's excited to conduction band at T - can be accelerated by **E** - holes also - Semiconductor (Si)
- Intrinsic & doped(n and p)

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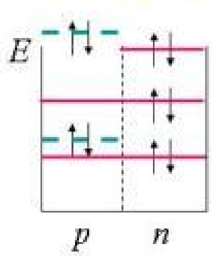
10. Nuclear properties

Nuclear properties

- Z=atomic number: Nuclide
  A=N+Z= mass no.: Isotope ($_1{}^1$H, $_1{}^2$H=d, $_1{}^3$H=t)
  $m_n > m_p$ (0.2%) : Nucleons
- Size: $R=R_0A^{1/3}$ (R_0~1.5 fm, fm=femtometer=10^{-15}m)
- Strong (or Hadronic) Force ~ same for n & p
- N ≥ Z : Coulomb repulsion elevates proton levels



n&p are bound in Strong Nuclear Potential
Nuclear quantum states and Ψ
For higher Z next n levels will be considerably lower in E than next p levels - e.g. $_{92}{}^{238}$U

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11. Nuclear properties II

Nuclear properties II

- Binding Energy $E_{BE}=(Zm_p+Nm_n-M_A)c^2$
  - Mass defect, curve of binding energy, fission, fusion
- Magnetic moments: nucleons first

$$\mu_{Nuclear} = \frac{e\hbar}{2m_p} = 3.15\times10^{-8}\ \text{eV / Tesla}\ \ (\text{cf. } 5.79\times10^{-5})$$

$$\mu^p{}_Z = +2.79\mu_N \qquad \mu^n{}_Z = -1.91\mu_N$$

- Nuclear Magnetic Moments - combinations
- Nuclear Magnetic Resonance and MRI scans

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12. Nuclear decays

Nuclear decays

- α: $(Z,A)\rightarrow(Z-2,A-4)+{}_2^4He$
  - Strong or nuclear glue and quantum tunneling
- β: $(Z,A)\rightarrow(Z+1,A)+e^-$ also have $(Z-1,A)+e^+$
  - Weak and based on $n\rightarrow p+e^-+\nu$ *(anti-neutrino)*
- γ: $(Z,A)^*\rightarrow(Z,A)+\gamma$
  - EM quantum transition
- Each with characteristic decay time and energy release (Q value)

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13. Radioactivity

Radioactivity

- All decays occur randomly - probability distribution characteristic of nuclide and decay products
- Activity=rate of decrease in time=-dN/dt=$\underline{A}$(t)
- $\underline{A}$ in units of Curies=3.7×10^{10} disintegrations/sec or Becquerels (Bq) SI units
- $\underline{A} \propto$ N(t) (each decay is independent of others) with $\underline{A} = \lambda N(t)$ λ=decay constant
- How does N change in time?

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14. Exponential decay

Exponential decay

$$\underline{A}(t) = -\frac{dN}{dt} = \lambda N(t) \qquad \text{So} \quad \frac{dN}{N} = -\lambda\, dt$$

$$\text{and} \int_{N_0}^{N(t)} \frac{dN}{N} = -\int_0^t \lambda\, dt \qquad \text{or} \quad \ln(N(t)) - \ln(N_0) = -\lambda t$$

$$\text{or} \quad N(t) = N_0 e^{-\lambda t} \quad \text{and} \quad \underline{A}(t) = \underline{A}_0 e^{-\lambda t}$$

$$\text{Hence} \quad \frac{1}{\lambda} = \tau \quad \text{mean lifetime}$$

Note also that $\underline{A}(t)$ will be $\frac{1}{2}\underline{A}_0$ for $e^{-\lambda t} = \frac{1}{2}$ or $t_{1/2} = \frac{\ln 2}{\lambda}$

$t_{1/2}$ is the half-life. Exponential decay law for many phenomena, especially QM decays and heat transfer.

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15. Decay examples

Decay examples

NUCLEUS	ATOMIC WEIGHT	HALF LIFE	DECAY PRODUCTS
Ra	226	1602 years	alpha,gamma(4%)
C	14	5730 years	beta
Sr	90	28 years	beta
^{3}H	3	12.3 yr	beta
I	131	8 days	beta,gamma
Cs	137	30 yr	beta,gamma
Rn	222	3.8 days	alpha,gamma(0.1%)
U	238	4.50 x 10^9 yr	alpha,gamma(23%)
Pu	239	24,400 yr	alpha

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