1. Physics 13: Electron spin

Physics 13: Electron spin

- Pauli - Goudsmit and Uhlenbeck - electron has intrinsic spin

$$|\vec{S}| = \sqrt{s(s+1)}\hbar \text{ with } s = \frac{1}{2} \text{ and } m_s = \pm\frac{1}{2}$$

s = 1/2 half integer spin
2s+1=2 multiplicity

e spin gets $\mu_z^{(s)} = -g_s m_s \mu_B$ with g_s gyromagnetic ratio
$g_s = 2.00232$

$\mu^{(s)}$ can interact with B field also
need extra label for $\psi_{n,l,ml,ms}$ and extra states

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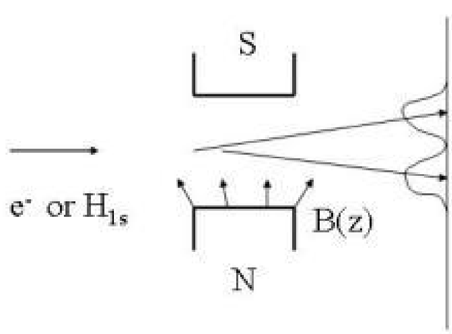
2. Stern-Gerlach experiment

Stern-Gerlach experiment



$$F_z = \mu_z \frac{dB_z}{dz} = \pm\mu_B \frac{dB_z}{dz}$$

Get 2 lines on the screen for m_s=+1/2 and - 1/2

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3. QM Angular momentum addition

QM Angular momentum addition

$$\vec{J} = \vec{L} + \vec{S}$$

Any QM angular momentum is quantized

$$\left|\vec{L}\right| = \sqrt{l(l+1)}\hbar, \quad \left|\vec{S}\right| = \sqrt{s(s+1)}\hbar$$

$$\left|\vec{J}\right| = \sqrt{j(j+1)}\hbar$$

j must be integer or half-integer

Rule: $j = l - s,\ l - s + 1,\ \ldots,\ l + s$

What does this imply about angular momentum vectors?

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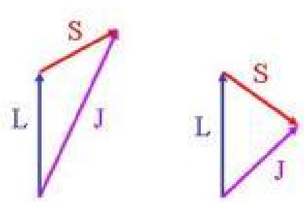
4. Angular momentum addition example

Angular momentum addition example

e.g. $l = 1,\ s = \frac{1}{2} \rightarrow j = \frac{3}{2}$ or $\frac{1}{2}$

$$L = \sqrt{2}\hbar,\ S = \frac{\sqrt{3}}{2}\hbar,\ J = \frac{\sqrt{15}}{2}\hbar \text{ or } \frac{\sqrt{3}}{2}\hbar$$



Only 2 allowed configurations that satisfy magnitude constraints

In general: $\vec{J} = \vec{J}_1 + \vec{J}_2 \rightarrow j = j_1 + j_2,\ j_1 + j_2 - 1,\ \ldots,\ \left|j_1 - j_2\right|$

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5. Total angular momentum

Total angular momentum

In atom, single e^- states have $n\, l\, m_l\, m_s$ (4th q.no.- others)
or $n\, j\, l\, m_j$
where $j = l \pm 1/2$ $\quad$ $n\,L_J$ notation:

$1S_{1/2}$, $2S_{1/2}$, $2P_{1/2}$, $2P_{3/2}$, $3S_{1/2}$, $3P_{1/2}$, $3P_{3/2}$, $3D_{3/2}$, $3D_{5/2}$,
… , $4F_{5/2}$, $4F_{7/2}$, … , $5G_{7/2}$, $5G_{9/2}$ …

These designations are appropriate for Fine Structure
(depends on $\boldsymbol{L \bullet S}$ or spin-orbit forces)

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6. Spin splittings

Spin splittings

Recall from E&M $U = -\vec{\mu} \bullet \vec{B} = -\mu_z B$ (for $\hat{z}$ along $\vec{B}$)
so min energy for $\vec{\mu}$ parallel to $\vec{B}$

- External ***B*** splits different m_J values - Zeeman
- Internal ***B*** due to ***L*** or in e^- frame: +Ze circles around → current loop and $\boldsymbol{B}_{\text{internal}}$ → $\boldsymbol{B}_{\text{internal}}$ parallel to ***L***
- $\mu_S \propto -\boldsymbol{S}$ for e^- so minimum energy *U* for ***L*** and ***S*** anti-parallel

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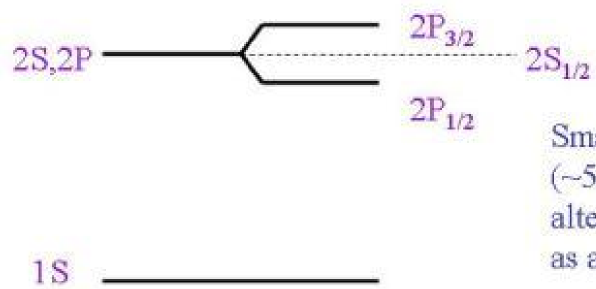
7. Fine structure splitting in H

Fine structure splitting in H

- Hence $j = l + s = l + 1/2$ will have higher energy than $j = l - s = l - 1/2$ states



Small splitting ($\sim 5\times10^{-5}$ eV) alters H spectrum as actually seen

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8. Other atoms with more e- ‘s

Other atoms with more e^- 's

In QM: system of particles described by single wave function for all the classical position variables

e.g. $x_1(t)$ and $x_2(t)$ $\rightarrow$ $\Psi(x_1, x_2, t)$

stationary equation for 2 identical particles:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}\psi(x_1,x_2) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}\psi(x_1,x_2) + U(x_1,x_2)\psi(x_1,x_2) = E\psi(x_1,x_2)$$

But which is particle 1 and which is particle 2?

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9. Indistinguishability

Indistinguishability

QM: identical particles are **indistinguishable**
(all electrons are identical, all protons, …)

Probability density: $|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$

So $\psi(x_1, x_2) = +\psi(x_2, x_1)$ symmetric under interchange

or $\psi(x_1, x_2) = -\psi(x_2, x_1)$ anti-symmetric under interchange

Only 1 state. For latter can not have two particles at same x

Profound consequences!

Two kinds of particles:

- bosons - satisfy Bose-Einstein statistics - symmetric
- fermions - satisfy Fermi-Dirac statistics - anti-symmetric

10. Identical particles

Identical particles

For 2 identical **non-interacting** particles $U(x_1,x_2) = U(x_1)+U(x_2)$
and Schrödinger equation separates so $\psi(x_1,x_2) = \psi(x_1)\ \psi(x_2)$
e.g. 1 dim. box

$$\psi_{n_1}(x_1)\psi_{n_2}(x_2) = A\sin\left(\frac{n_1\pi}{2L}x_1\right)\sin\left(\frac{n_2\pi}{2L}x_2\right)$$

$$\rightarrow E = E_{n_1} + E_{n_2} = \frac{\hbar^2 k_1^2}{2m} + \frac{\hbar^2 k_2^2}{2m}$$

Symmetrizing:

$$\psi_{S\ or\ A}(x_1, x_2) = A'\left(\psi_{n_1}(x_1)\psi_{n_2}(x_2) \pm \psi_{n_1}(x_2)\psi_{n_2}(x_1)\right)$$

For **anti**-symmetric case can **not** have $n_1 = n_2$

11. Particle statistics

Particle statistics

- Bosons - integer spin - photon,W, Z, graviton,integer spin atoms, spinless objects
  - Bose-Einstein Condensate
- Fermions - half-integer spin - *e*, *p*, *n*, μ, quarks, neutrinos
  - Pauli Exclusion Principle
    - for atomic electrons can have 2 in same ($n\ l\ m_l$) providing they have different m_s

    i.e. opposite *spin projection* - *up and down*
- Entanglement

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12. Building atoms- Periodic Table

Building atoms- Periodic Table

- Ignore electron+electron repulsion as first approximation
- Orbitals (*l* - values) fill in $m_S=\pm 1/2$ pairs
- Shells: K (n=1), L (n=2), M (n=3), . . .

$_2$He (Z=2) 1s ($m_S=+1/2$) AND 1s ($m_S=-1/2$) or $1s^2$ represents the ground state configuration

- What is the ground state energy?

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13. Helium energy levels

Helium energy levels

- Single e^- $E_{\text{ground state}} = - Z^2 E_0 = -54.4$ eV
- Second e^- *"sees"* shielded charge or repulsion from 1st e^-. How does this change the E for one e^- ?
- *Inonization Energy* is 24.6 eV (tightly bound $\Rightarrow$ inert gas)
- So repulsion of 54.4 - 24.6 eV = 29.8 eV

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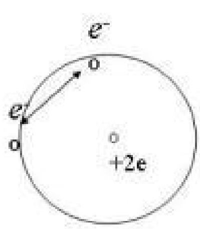
14. He repulsion

He repulsion



Repulsive energy - average

$$\langle U \rangle = \frac{k_{EM} e^2}{\langle r \rangle_{separation}} = 29.8 \text{ eV}$$

$$\rightarrow \langle r \rangle_{separation} = 0.456\, a_0$$

Two e^-'s have opposite m_s (+1/2, -1/2) and are in the same space state which gives this pair extra stability (c.f. $1s^1 2s^1$)

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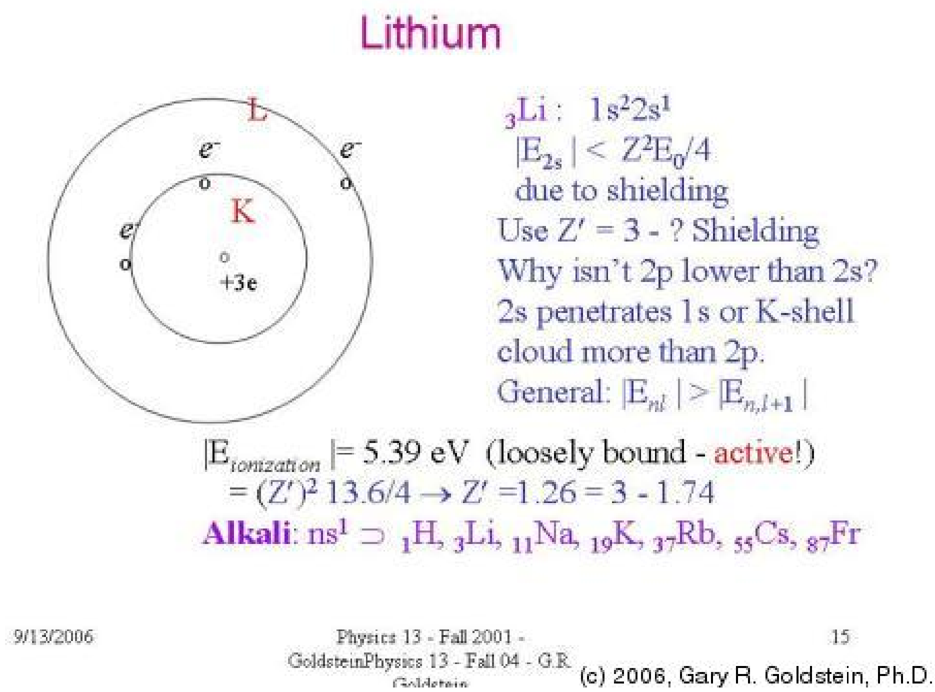
15. Lithium



16. Periodic Table