

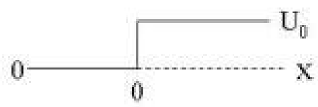
1. Physics 13 – Steps and barriers

Physics 13 – Steps and barriers

Consider stationary plane waves ~exp(ikx)

- Step potential: U(x)=0 for x<0

  $=U_0$ for x>0



impulsive force $-U_0/\Delta x$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x)+U(x)\psi(x)=E\psi(x)$$

Time independent 1-dim Schrödinger equation

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 1

(c) 2006, Gary R. Goldstein, Ph.D.

2. Lecture 11: Steps, Tunneling, Wells: Slide 2

Barrier penetration

For $x<0 \rightarrow \frac{\hbar^2 k_1^2}{2m}=E$ or $k_1=\frac{\sqrt{2mE}}{\hbar}$ and $\psi \propto e^{ik_1 x}$

But for $x>0 \rightarrow \frac{\hbar^2 k_2^2}{2m}=E-U_0$ 2 cases

$k_2=\frac{\sqrt{2m(E-U_0)}}{\hbar}$ for $E>U_0$ and $\psi \propto e^{ik_2 x}$

or $k_2=\frac{i\sqrt{2m(U_0-E)}}{\hbar}$ for $E<U_0$ and $\psi \propto e^{-|k_2|x}$

Note the exponential falloff for 2nd case
Need reflected waves to match at x=0

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 2

(c) 2006, Gary R. Goldstein, Ph.D.

3. Barrier wave functions

Barrier wave functions

So for $E > U_0$ at $x < 0$ can write $\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$

at $x > 0$ have $\psi(x) = Ce^{ik_2x}$ forward moving

Matching $\psi(0)$ and $d\psi/dx|_0$ relates A, B to C
B/A = $(k_1-k_2)/(k_1+k_2)$ and A=C $(k_1+k_2)/2k_1$
Reflection probability is $(B/A)^2$ =R and
Transmission coefficient is T=1-R

For $E < U_0$ at $x < 0$ can write $\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$

but at $x > 0$ have $\psi(x) = Ce^{-|k_2|x}$ forward decaying

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 3

4. Barrier waves in time

Barrier waves in time

Multiply ψ by $e^{-i\omega t}$ which turns each term into travelling plane wave, A and C parts to the right, B to the left - reflected

For decaying case partial penetration into barrier as allowed by uncertainty in $\Delta x\ \Delta p$.

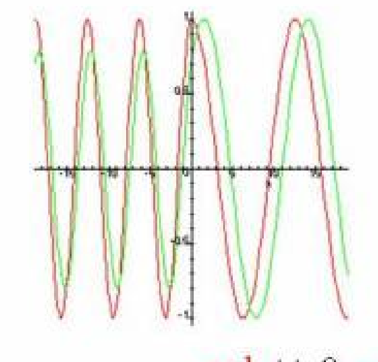
Quantum Scattering demo of traveling waves & barriers or wells

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 4

5. Barrier wave function graph

Barrier wave function graph

Example: $U_0=3E/4$ so $k_2=k_1/2$ or $\lambda_2=2\lambda_1$



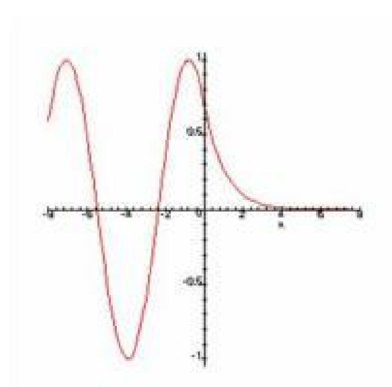
red at t=0, green at $\omega t=\pi/4$

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 5

6. Barrier penetration graph

Barrier penetration graph

$E<U_0$ so get particle under barrier

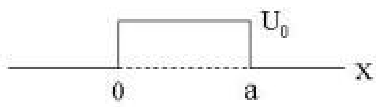


9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 6

7. Finite barriers

Finite barriers

U(x)=0 for x<0 and x>a
=U_0 for 0<x<a
impulsive force $-U_0/\Delta x$ and $+U_0/\Delta x$



For $E < U_0$ at $x < 0$ and $x > a$ can write $\psi(x) = Ae^{\pm ik_1 x}$

but for $0 < x < a$ have $\psi(x) = Ce^{\mp\alpha x}$

where $\alpha = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 7

8. Tunneling

Tunneling

Join solutions at *x=0* and *a* *(complicated)*
For αa>>1 have simpler transmission
T≈ exp(-2 αa) rapid falloff

Tunneling through a barrier example:
α-decay of nuclei
Scanning-tunneling electron microscope
Josephson junction
Cold fusion??

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 8

9. Expectation values

Expectation values

Side issue:
Recall that average of x^2 for the square well was given by

$$\langle x^2 \rangle_n = \int_0^L dx \left(x^2 \frac{2}{L} \sin^2(n\pi x / L) \right)$$
$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^{3} \int_0^{n\pi} d\vartheta (\vartheta \sin\vartheta)^2 = L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right)$$

This was the expectation value of x^2 for the n^{th} energy state in the well.

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 9

10. General expectation values

General expectation values

- Consider system having some potential $U(x)$
- There will be a set of stationary solutions to the Schrödinger equation- wave functions $\psi_n(x)$ (or states) - corresponding to E_n 's
- For properly normalized $\psi_n(x)$, the expectation value of a function $f(x)$ in state n

$$\langle f(x) \rangle_n = \int_{-\infty}^{+\infty} f(x) |\psi_n(x)|^2 dx$$

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 10

11. Finite well

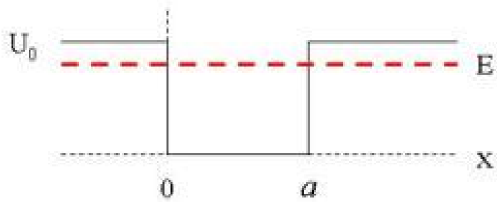
Finite well

$U(x) = U_0$ for $x<0$ and $x>a$

$=0$ for $0<x<a$

impulsive force $+U_0/\Delta x$ and $-U_0/\Delta x$ at $x=0$ and a

With $E < U_0$ the particle is trapped



9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 11

12. Finite well equations

Finite well equations

Recall the time independent 1-dimensional Schrödinger equation

stationary states: $\Psi(x,t) = \psi(x)e^{-i\omega t}$ with $E = \hbar\omega$

$$\rightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + U(x)\psi(x) = E\psi(x) \text{ ordinary diff.eq.}$$

Boundary conditions for physical solution:

$\psi(x)$ is continuous and $\frac{d\psi(x)}{dx}$ is also for $U(x)$ finite.

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 12

13. Finite well solutions

Finite well solutions

$$\frac{d^2}{dx^2}\psi(x) = \frac{-2mE}{\hbar^2}\psi(x) \qquad \text{for} \quad 0 < x < a$$

and $E = \dfrac{\hbar^2 k^2}{2m}$ is correct energy with

general solution of form $\psi(x) = A\sin(kx) + B\cos(kx)$

But outside of well the wave function is not zero

$$\frac{d^2}{dx^2}\psi(x) = \frac{+2m(U_0 - E)}{\hbar^2}\psi(x) \quad \text{for} \quad x < 0 \quad \text{and}\, x > a$$

Note the sign - so $\psi(x) \sim e^{\pm\alpha x}$ for $x<0$ or $x>a$

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 13 (c) 2006, Gary R. Goldstein, Ph.D.

14. Finite well solutions cont’d

Finite well solutions cont'd

where $\alpha^2 = \dfrac{+2m(U_0 - E)}{\hbar^2} > 0$

For $\psi(x) \sim e^{\pm\alpha x}$

Have to satisfy continuity of $\psi(x)$ and $d\psi(x)/dx$
at $x=0$ and $x=a$

Only certain discrete values of k or E will work

Depends on U_0 and a

Quantized energies -- a property of bound systems

9/13/2006 Physics 13 - Fall 2001 - GoldsteinPhysics 13 - Fall 04 - G.R. Goldstein 14 (c) 2006, Gary R. Goldstein, Ph.D.