


1. Physics 13 – Wave Functions, Probability

Physics 13 – Wave Functions, Probability

- A wave needs a wave equation
- **Schrödinger Equation** (1928) wave  mechanics
 - (cf. Heisenberg, Born, Jordan matrix mechanics)
- $\Psi(x,t)$ wave function or amplitude- solution
- **complex** valued function
- Interpretation? cf. strings, sound, water waves
- Consider EM waves and **wave-particle duality**

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2. Schrödinger Equation (in 1 dimension)

Schrödinger Equation (in 1 dimension)

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) + U(x)\Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$$

- $\Psi(x,t)$ is complex and i appears on right
- 2nd order linear differential equation in 2(4) variables
- $U(x)$ is potential function (e.g. mgx , $-k_{EM}e^2/x$, $(1/2)kx^2$)

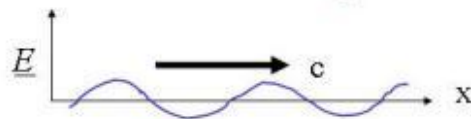
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3. EM wave-particle duality

EM wave-particle duality



- Maxwell: Energy/vol $\sim \underline{E}^2$ (or $\underline{E}^2 + \underline{B}^2$)
- But quanta have $E = h\nu$. How to reconcile?
 - Note that 100 W light bulb \rightarrow 30 W visible light with peak λ near 500 nm,
 - so $\nu = c/\lambda = 6 \times 10^{14} \text{ s}^{-1}$
 - and each γ has $E = 6.6 \times 10^{-34} \text{ J}\cdot\text{s} \cdot 6 \times 10^{14} \text{ s}^{-1}$
 - $\sim 4 \times 10^{-19} \text{ J}$.
 - Hence no. γ 's $\sim 30 \text{ J/s} / 4 \times 10^{-19} \text{ J}$
 - $\sim 8 \times 10^{19} \gamma/\text{sec}$

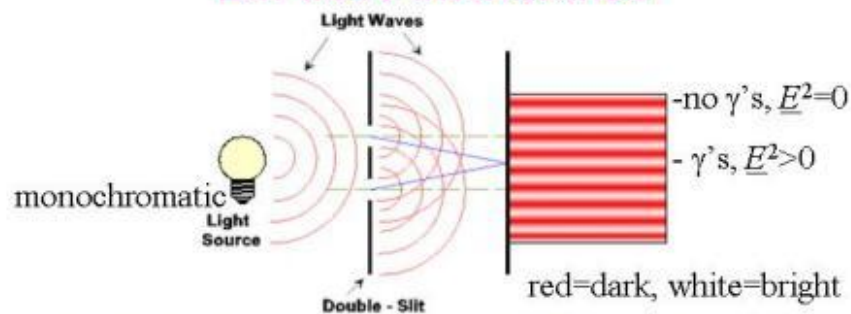
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4. EM wave interference

EM wave interference



- $\underline{E} = \underline{E}_1 + \underline{E}_2$ and $|\underline{E}|^2 \neq |\underline{E}_1|^2 + |\underline{E}_2|^2$
- As $|\underline{E}|^2$ (\sim Probability for γ 's) decreases to few γ 's get same pattern *on the average*

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5. Light, bb's, electrons

Light, bb's, electrons

- Compare EM interference with bb's shot through 2 slits ($\rho_{\text{probability}} = \rho_1 + \rho_2$)
- Electrons are like light, not bb's.
- $\Psi = \Psi_1 + \Psi_2$ and $\rho \propto |\Psi|^2 = |\Psi_1 + \Psi_2|^2 \neq |\Psi_1|^2 + |\Psi_2|^2$
- $\Psi(\underline{x}, t)$ is the **amplitude** for an electron at \underline{x}, t .
- $|\Psi(\underline{x}, t)|^2$ is Probability Distribution Function or Probability Density, with $|\Psi(\underline{x}, t)|^2 \Delta V$ the **Probability for finding e^- in Volume ΔV**

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6. Normalizing probability

Normalizing probability

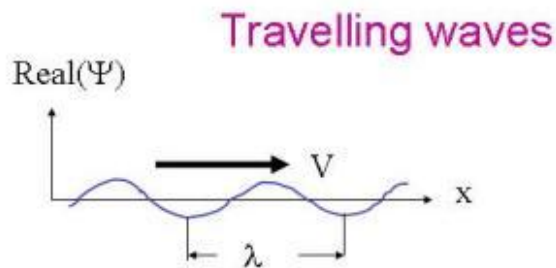
- Necessary mathematical requirement for physical sense
- $\rho(\underline{x}, t) = |\Psi(\underline{x}, t)|^2$ in 3-dim space
- In 1-dim $\rho(x, t) = |\Psi(x, t)|^2$ and $\rho(x, t) dx$ is probability in length dx
- So need $\int_{\text{All } x} dx \rho(x, t) = 1$ since single electron is **somewhere**
- This is normalization of probability and amplitude Ψ

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7. Travelling waves



- Time development of Ψ is the dynamics in QM
- Travelling plane wave (1-dim) is a solution for a **free particle**:

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8. Plane wave variables

Plane wave variables

$$\Psi(x, t) = A \sin(kx - \omega t) \quad \text{where } k = \frac{2\pi}{\lambda} \quad \text{and } \omega = 2\pi\nu$$

for a travelling plane wave.

But for Quantum Physics $\lambda = \frac{h}{p}$ so $\hbar k = p$.

Also $\frac{p^2}{2m} = E \rightarrow \hbar\omega = \frac{\hbar^2 k^2}{2m}$ dispersion

velocity $V = \lambda\nu = \frac{\omega}{k}$ depends on k ($= \frac{1}{2} \frac{p}{m}$)

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9. Interpretation of plane wave

Interpretation of plane wave

- Infinite in extent \Rightarrow completely unlocalized particle with definite wavelength & frequency (for precise spatial shape & time dependence) or momentum & energy
- To localize need **Fourier Superposition**
- Consider 2 waves - both solutions (linearity allows superposition)

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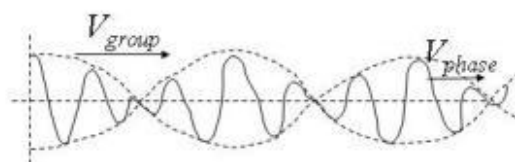
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10. Velocities

Velocities

$$\begin{aligned} \Psi(x,t) &= A_0 \sin(k_1 x - \omega_1 t) + A_0 \sin(k_2 x - \omega_2 t) \\ &= 2A_0 \cos\left(\frac{k_1 - k_2}{2} x - \frac{\omega_1 - \omega_2}{2} t\right) \sin\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right) \\ &= [2A_0 \cos\frac{1}{2}(\Delta k \cdot x - \Delta \omega \cdot t)] \sin(k_{ave} x - \omega_{ave} t) \\ \text{envelope: } V_{group} &= \frac{\Delta \omega}{\Delta k} \text{ and } V_{phase} = \frac{\omega_{ave}}{k_{ave}} \end{aligned}$$



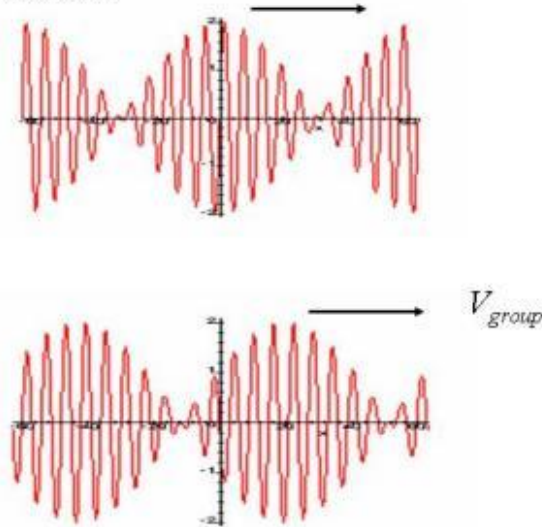
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11. Lecture 9: Wave Functions, Probability: Slide 11

Illustration



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12. Phase relations

Phase relations

Note that envelope = 0 at neighboring points x_1, x_2

such that $\frac{1}{2} \Delta k \cdot x_2 - \frac{1}{2} \Delta k \cdot x_1 = \pi$ at fixed t

or $\Delta k \Delta x = 2\pi$ ($\Delta x = x_2 - x_1$)

For fixed x get similar relation for neighboring times t_1, t_2

$$\Delta \omega \Delta t = 2\pi$$

Not a finite wave packet yet. Need Fourier series:

$$\sum_j A_j \sin(k_j x - \omega_j t)$$

Gets closer, but still not completely localized (periodic). Need continuum limit or Fourier Integral.

$$\rightarrow \int_0^\infty A(k) \sin(kx - \omega t) dk \quad [\omega = \omega(k)]$$

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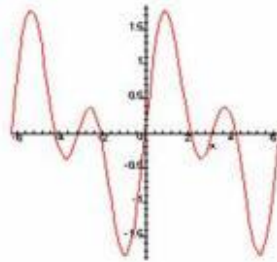
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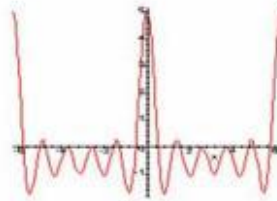
13. Fourier series

Fourier series

$\sin(x) + \sin(2x) \rightarrow$



$\cos(x) + \cos(2x) + \dots + \cos(5x) \rightarrow$



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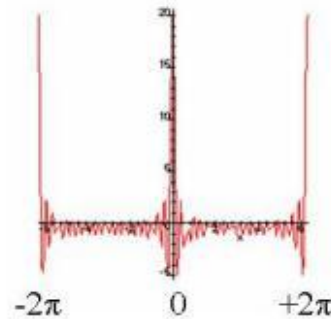
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14. Fourier series cont'd

Fourier series cont'd

$\sum_{n=1}^{20} \cos(nx) \rightarrow$



Very localized, but still periodic. Changing scale will push recurrences farther away. $nx \rightarrow (n/10)x$

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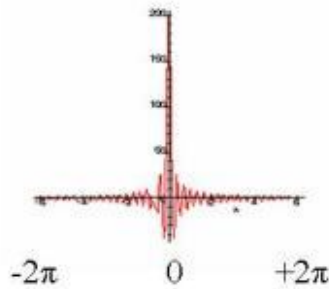
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15. Fourier series scale change

Fourier series scale change

$$nx \rightarrow (n/10)x$$

$$\sum_{n=1}^{200} \cos\left(\frac{n}{10}x\right) \rightarrow$$



Repeated pattern at $\pm 20\pi$.

By letting $n/10 \rightarrow k$ (continuous) get integral over dk .

This example gives all k equal weight.

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16. Fourier coefficients

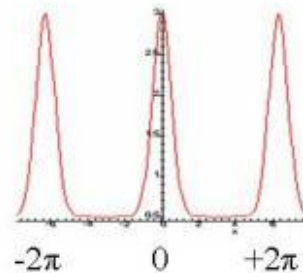
Fourier coefficients

$$\sum_{n=0}^{\infty} A_n \cos(nx) \rightarrow \text{any periodic even function}$$

$$A_n = e^{-\frac{n^2}{2(\Delta n)^2}} \quad \text{smoothes out oscillations}$$

Gaussian weighting.

For $\Delta n=2 \rightarrow$



This A_n weights n values around 2 and below.

$n \gg 2$ is highly suppressed. Or $n \gg \Delta n$.

Increasing Δn narrows the peak (as it allows more cancellations among higher n waves).

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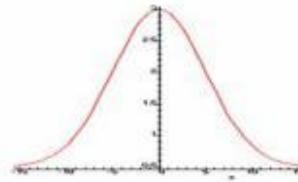
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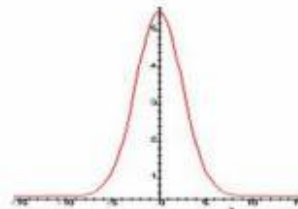
17. Fourier series scale

Fourier series scale

Change scale by $n \rightarrow n/10$
and $\Delta n = 2 \rightarrow 2/10$



Similarly $n \rightarrow n/10$
and $\Delta n = 4 \rightarrow 4/10$



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18. Fourier integral

Fourier integral

As $n/N \rightarrow$ continuum or k
sum becomes integral

$$A_n \rightarrow A(k)$$

Integral will be even function of x

For gaussian with width Δk

$$\int_0^{\infty} A(k) \cos(kx) dk$$

$$A(k) = e^{-\frac{k^2}{2(\Delta k)^2}} / (\Delta k \sqrt{\pi/2}) \text{ is normalized to 1}$$

$$\text{so integral yields } e^{-\frac{\Delta k^2 x^2}{2}} \text{ or } e^{-\frac{x^2}{2(\Delta k)^2}}$$

Corresponding width in x , $\Delta x = 1/\Delta k$

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19.

Heisenberg Uncertainty Principle

Heisenberg Uncertainty Principle

- In general: Fourier integral of distribution $A(k)$ of width Δk is function in x of width Δx with $\Delta x \Delta k \geq 1/2$

- Then for QM waves, where

$$p = \hbar k \text{ or } k = p / \hbar$$

$$\Delta x \cdot \Delta p \geq \hbar / 2 \quad \text{Heisenberg Uncertainty Principle}$$

Similar considerations in time dependence

with ω ($E = \hbar\omega$) and t gives

$$\Delta t \cdot \Delta E \geq \hbar / 2 \quad \text{Philosophical implications}$$

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