

1. Physics 13: Bohr Atom & Electron Waves

Physics 13: Bohr Atom & Electron Waves

Atomic Spectra

- Each element has characteristic discrete EM spectral lines (1880's)
- H (& single e atoms) Rydberg-Ritz formula
  $1/\lambda = RZ^2(1/n_2^2 - 1/n_1^2)$ for $n_1 > n_2$
  $R_\infty = 10.97373\ (\mu m)^{-1}$ Rydberg constant(for heavy elements)
- Rutherford atom ruled out Thompson's "plum pudding" model and supported "solar system" model
  Atom $r \sim 10^{-9 \text{ or } -10}$m but nucleus $r \sim 1$ fm or 10^{-15} m
- Why don't orbiting electrons radiate?

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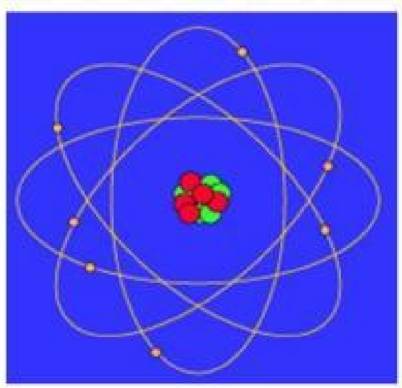
2. Models of the atom

Models of the atom



Planetary model - electrons around nucleus
Why don't electrons radiate as Maxwell predicts?

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3. Bohr’s H atom

Bohr’s H atom

$F_{Coulomb} = k_{EM}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$ for single orbiting electron

Hence $mv^2 = k_{EM}\frac{Ze^2}{r}$ $(k_{EM}=1/4\pi\varepsilon_0)$

Total e Energy $E = \frac{1}{2}mv^2 - k_{EM}\frac{Ze^2}{r}$

or $E = -\frac{1}{2}mv^2 = -\frac{1}{2}k_{EM}\frac{Ze^2}{r}$ **Non-relativistic**

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4. Bohr’s quantum condition

Bohr’s quantum condition

Angular momentum is quantized $mvr = n\frac{h}{2\pi} = n\hbar$

where $n = 1, 2, \ldots$

squaring $m^2v^2 = \frac{n^2\hbar^2}{r^2}$, so $mv^2 = \frac{n^2\hbar^2}{mr^2}$.

Substituting for mv^2 yields $\frac{n^2\hbar^2}{mr^2} = k_{EM}\frac{Ze^2}{r}$

Then allowed orbits have $r_n = \frac{n^2\hbar^2}{k_{EM}Ze^2m} = \frac{n^2}{Z}a_0$

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5. Quantized orbits

Quantized orbits

Bohr radius $a_0 = \frac{\hbar^2}{k_{EM} e^2 m} = 5.29 \times 10^{-11} m = 0.529$ A

Substitute for r in E: $E_n = -\frac{1}{2} k_{EM} Z \frac{e^2}{r_n} = \frac{k_{EM}{}^2 e^4 m Z^2}{2\hbar^2 n^2}$

or $E_n = -\frac{Z^2}{n^2} E_0$ with $E_0 = 13.6$ eV

So $-E_0$ is the lowest Energy " state"

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6. Radiative transitions

Radiative transitions

Quantum jump occurs

electron jumps from orbit i to f which releases photon

$h\nu = E_i - E_f$

then $\nu = \frac{Z^2 E_0}{2\pi\hbar}\left(\frac{1}{n_f{}^2} - \frac{1}{n_i{}^2}\right)$ Ryderg formula!

and $R_\infty = E_0 / h$ which is in exact agreement

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7. Transition example

Transition example

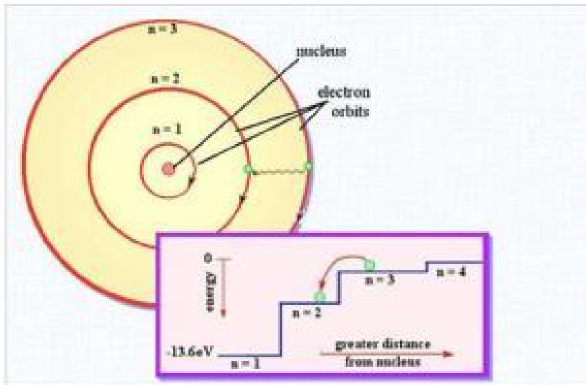

Image adapted by M. Nguyen

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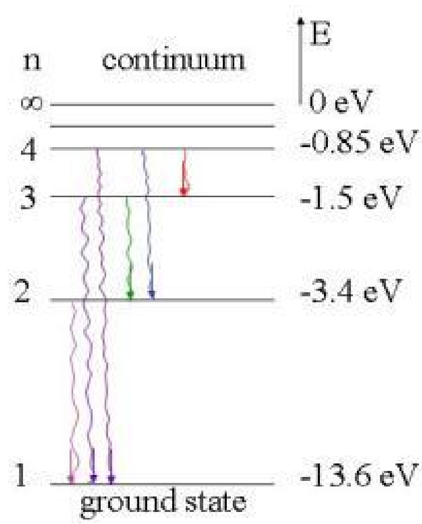
8. Energy levels of H

Energy levels of H



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9. Spectral lines of H

Spectral lines of H

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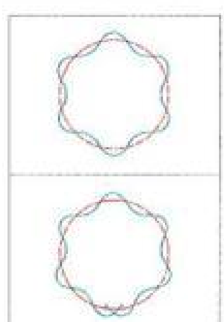
10. deBroglie waves

deBroglie waves

- Why does the quantization condition work?
- L. deBroglie (1924): electron has wave properties! Orbits have standing waves.



$2\pi r/\lambda$ = integer

$2\pi r/\lambda \neq$ integer

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11. Particle-wave duality

Particle-wave duality

- What is the λ for e^- ?
- Analogy with photon: $E=h\nu$ and $E=pc$
- So $\lambda=c/\nu= hc/h\nu=h/p$ deBroglie uses this
- deBroglie wavelength of massive particle

$\lambda = h/p$ (but $pc \neq E$ for $m_0 \neq 0$)

Ex. $m_{baseball}=0.17$ Kg, $v=100$ Km/hr $\rightarrow \lambda=1.4\times10^{-34}$m

$m_{proton}=1.7\times10^{-27}$Kg, $v=c/100 \rightarrow \lambda=1.3\times10^{-13}$m

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12. Electron waves

Electron waves

- e in H atom: $\lambda = h/p = h/(mv)$
  - Important: this is for **non-relativistic** motion
- But Bohr condition: $mvr = nh/(2\pi)$
- So $2\pi r = n\,h/(mv) = n\,\lambda$ just as needed
- Note that $p^2 = 2mK = 2mE_0$ for ground state
- So $\lambda_1 = h/\sqrt{(2mE_0)} = 1.226\times10^{-9}\text{m}/\sqrt{E_0}$
- $= 3.32\times10^{-10}\text{m} = 2\pi a_0$
- Also $|E_2|=E_0/4$ & $r_2=4a_0$, so $\lambda_2=2\lambda_1=(1/2)2\pi r_2$

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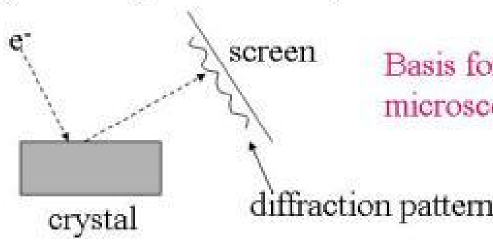
13. Electron wave effects

Electron wave effects

- e^- 's should show diffraction for material spacing same order as deBroglie wavelength
- Crystal diffraction - Davisson & Germer (1927) (analog of X-ray diffraction)



Basis for electron microscope

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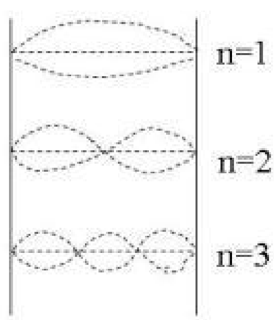
14. Standing e waves

Standing e waves



1 dim box - length L

$\lambda_1=2L,\ \nu_1=v/(2L)=v/\lambda_1$

$\lambda_2=L,\ \nu_2=v/(L)=v/\lambda_2$

$\lambda_3=2L/3,\ \nu_3=3v/(2L)=v/\lambda_3$

.

$\lambda_n=2L/n,\ \nu_n=nv/(2L)=v/\lambda_n$

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15. Electron Wave Function

Electron Wave Function

- A wave needs a wave equation
- Schrödinger Equation (1928) wave mechanics
  - (cf. Heisenberg, Born, Jordan matrix mechanics)
- $\Psi(\underline{x},t)$ wave function or amplitude- solution
- complex valued function
- Interpretation? cf.strings, sound, water waves
- Consider EM waves and wave-particle duality

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