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1. Physics 13 – Approaching General Relativity

Physics 13 – Approaching General Relativity

- Equivalence Principle
  - Behavior of objects in Constantly Accelerating frame is indistinguishable from falling objects in G field ("feels" like gravity - more or less)
  - Objects in a "free" falling frame (with G) are indistinguishable from objects in inertial frame
    - Orbiting astronauts, free fall simulators, jumping

True because gravity attracts ∝amount of matter $m_{gravitational}$
& object responds ∝amount of inertia $m_{inertial}$

$m_{inertial}=m_{gravitational}$ so $\underline{F}_G = m_g\underline{g} = m_i\underline{a}$

(assuming small velocities only for simplicity)

- Because of this, all falling objects with same initial conditions will have same trajectory in elevator in distant space with a=g or elevator sitting on the earth.

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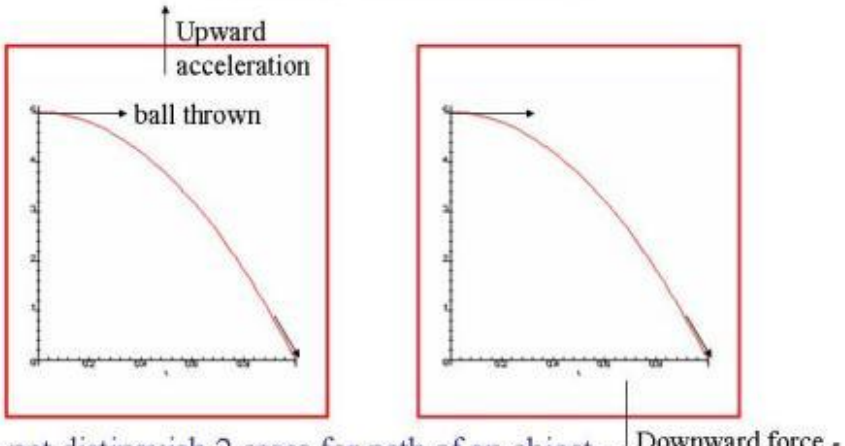
2. Lecture 6: General Relativity: Slide 2

Gravity or elevator?



Can not distinguish 2 cases for path of an object- gravity pulling on object vs. object seen in accelerating frame of reference

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3. Compare to other forces

Compare to other forces

- Consider constant $\underline{E}$ field acting on charged particles q_1 & q_2.
- Forces will be $q_1\underline{E}$ & $q_2\underline{E}$
- In inertial frame S: $\underline{a}_1 = q_1\underline{E}/m_1$ & $\underline{a}_2 = q_2\underline{E}/m_2$
- So can not get in single accelerating elevator S' without $\underline{E}$ that would cause both to follow same trajectory they had in S.
- Would have to have q_i/m_i be the same for all charged objects - it is not the case. **Gravity is very special!**

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4. Light paths

Light paths

- Path of light is same as path of "material object" so light "gravitates"
- Bending of light around massive objects
  - Coordinate system (space-time) depends on light paths to be established operationally
  - Massive objects bend the light paths so alter coordinate grid ⇒ alter geometry
  - To see how this happens consider rotating platform

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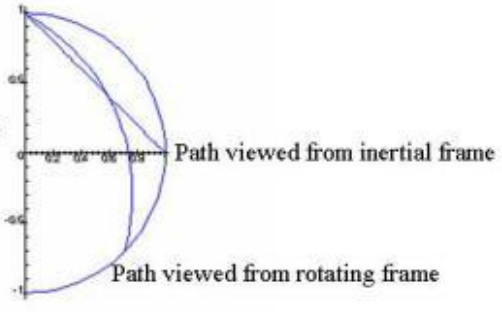
5. Straight path viewed in rotating frame

Straight path viewed in rotating frame

Rotation is counter-clockwise at a rate that moves 45° in the time it takes projectile to reach x-axis.



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6. Space-time curvature

Space-time curvature

- Space(-time) is curved by presence of massive objects
  - Consider meter sticks laid out along circumference of rapidly rotating turntable - circ≠2πr from either frame - inertial or non-inertial (stationary people or turntable dwellers)

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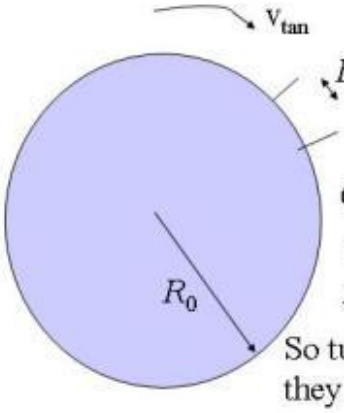
7. Non-rotating frame view

Non-rotating frame view



$L = L_0/\gamma$ (at R_0)

Measured L is shorter than turntable dweller

$C_0 = 2\pi R_0$ but $C = C_0 / \gamma(R_0)$

so $C < 2\pi R_0$ (positive curvature)

Also have $t(R_0) = t_0 \times \gamma(R_0)$ (red shift)

So turntable geometers will decide that they have non-Euclidian space (positive curvature).

Dweller near edge will feel pulled outward ~ outward pull of gravity

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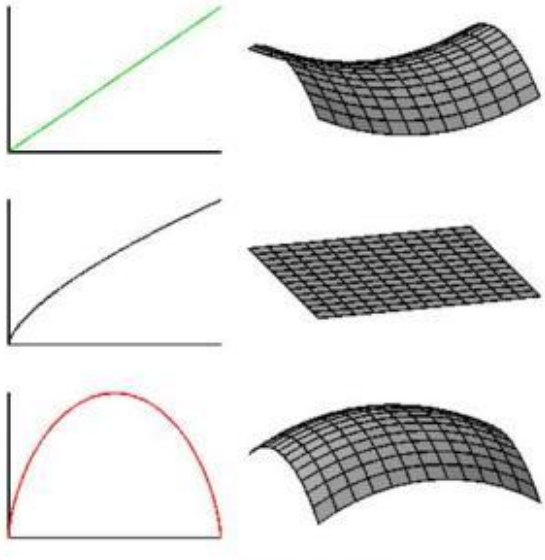
8. Curvature models (2-d in 3-d)

Curvature *models* (2-d in 3-d)



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9. Space-time curvature

Space-time curvature

- Space(-time) is curved by presence of massive objects
  - Consider meter sticks laid out along circumference of rapidly rotating turntable - circ≠2πr from either frame - inertial or non-inertial
  - Consider clocks on circumference - run slow - red shift
- Gravity *is* curved spacetime **General Relativity**
  - Elliptical orbits around Sun precess (precession of the perihelion of Mercury)

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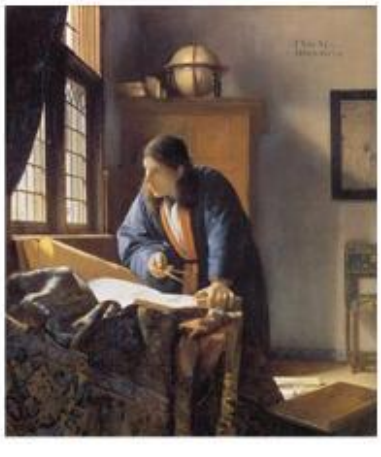
10. “Geometry is an experimental science”

"Geometry is an experimental science"



Vermeer - The Geographer 1668

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11. Gravitating light

Gravitating light

- Light has "gravitational mass" = inertial mass ($E/c^2 = p/c$ for Maxwellian EM wave = $h\nu/c^2$ in Quantum Physics)
- Light has gravitational potential energy along with "normal" energy ($h\nu$) leading to the gravitational red shift and black holes

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12. Frequency shifts

Frequency shifts

- $E=h\nu - GM(h\nu/c^2)/R = h\nu'$ for photon going from a star to free space or
  - $\nu' = \nu (1 - GM/Rc^2)$
- If $GM/Rc^2 > 1$ light can not emerge from strong gravitational field - **black hole** (Actually only need $GM/Rc^2 > 1/2$ or Schwarzchild radius -due to necessary synchronization of clocks in changing field.)

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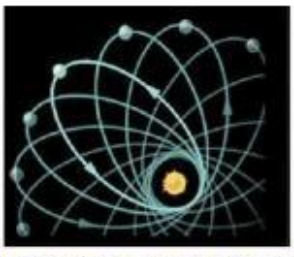
13. Consequences of General Relativity

Consequences of General Relativity

1. Elliptical orbits around Sun precess (precession of the perihelion of Mercury) due to curvature



2. Light has gravitational energy along with "normal" energy (via frequency or color), leading to the *gravitational red shift*
3. If gravity is very strong (very dense, large mass), light paths can not go outward - **Black Holes**

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14. Black hole

Black hole

- Consider light temporarily like "particle" with "mass" $E/c^2=m_\gamma$
- Throw particle from massive star, M & R at escape velocity v_{esc}
- Energy (Non-Relativistic) = KE+PE=0 for escape

$$\frac{1}{2}m_\gamma v^2_{esc} - \frac{GMm_\gamma}{R} = 0 \quad \text{so} \quad v^2_{esc} = \frac{2GM}{R}$$

$$\text{or} \quad \frac{v^2_{esc}}{c^2} = \frac{2GM}{Rc^2} \quad \text{Then } v_{esc} \to c \text{ yields } R_{blackhole} = \frac{2GM}{c^2}$$

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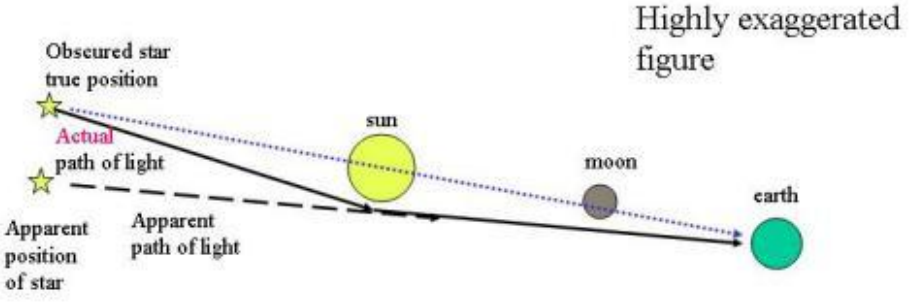
15. Consequences, cont’d

Consequences, cont'd

4. Bending of light & solar eclipse
Expedition of 1919 proved correct!



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16. Field equations

Field equations

Einstein: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$

R related to curvature of space-time
$g_{\mu\nu}$ related to metric
G Newton's gravitational constant coupling
$T_{\mu\nu}$ Energy-momentum of matter and radiation

Feynman's easier version for average curvature measure:

$$r_{excess} = \sqrt{\frac{Area}{4\pi}} - r_{measured} = \frac{GM}{3c^2}$$

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