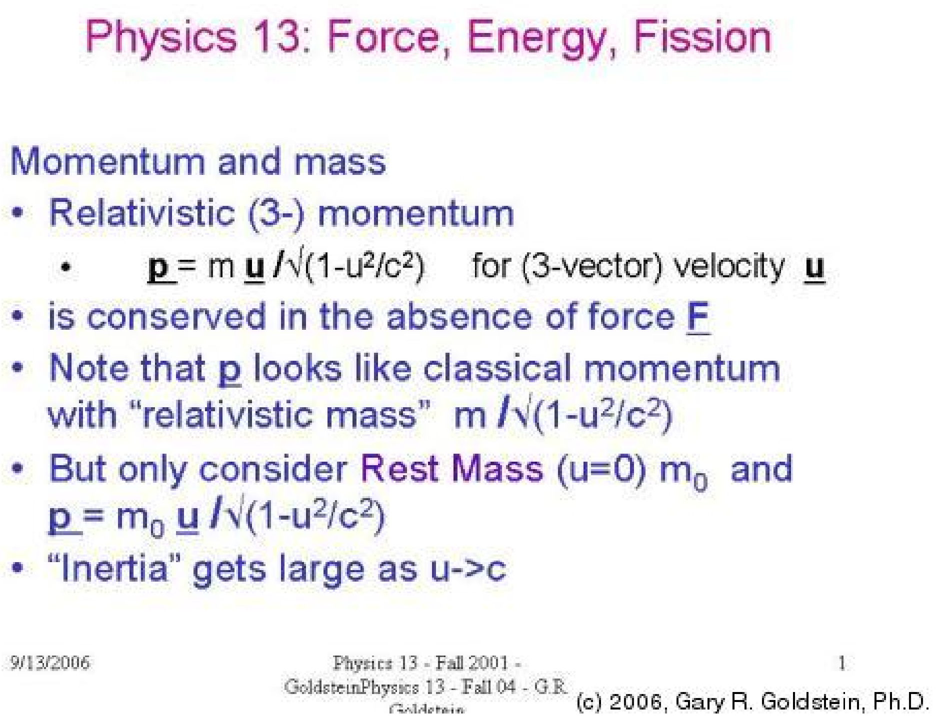
1. Physics 13: Force, Energy, Fission



Physics 13: Force, Energy, Fission

Momentum and mass

- Relativistic (3-) momentum
  - $\underline{p} = m\,\underline{u} / \sqrt{(1-u^2/c^2)}$ for (3-vector) velocity $\underline{u}$
- is conserved in the absence of force $\underline{F}$
- Note that $\underline{p}$ looks like classical momentum with "relativistic mass" $m / \sqrt{(1-u^2/c^2)}$
- But only consider Rest Mass (u=0) m_0 and $\underline{p} = m_0\,\underline{u} / \sqrt{(1-u^2/c^2)}$
- "Inertia" gets large as u->c

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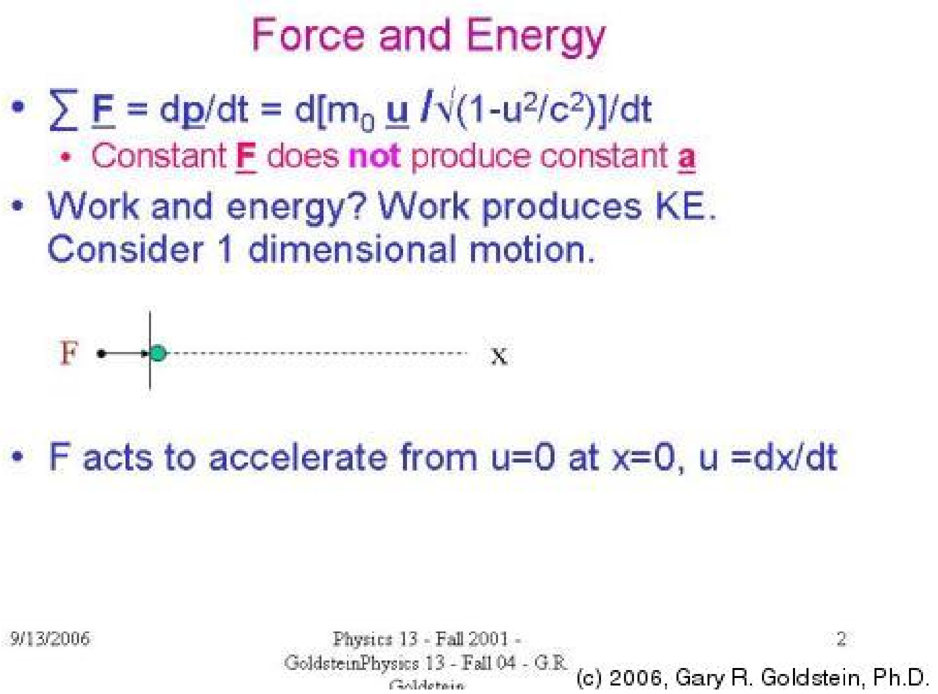
2. Force and Energy



Force and Energy

- $\sum \underline{F} = d\underline{p}/dt = d[m_0\,\underline{u} / \sqrt{(1-u^2/c^2)}]/dt$
  - Constant $\underline{F}$ does **not** produce constant $\underline{a}$
- Work and energy? Work produces KE. Consider 1 dimensional motion.

- F acts to accelerate from u=0 at x=0, u =dx/dt

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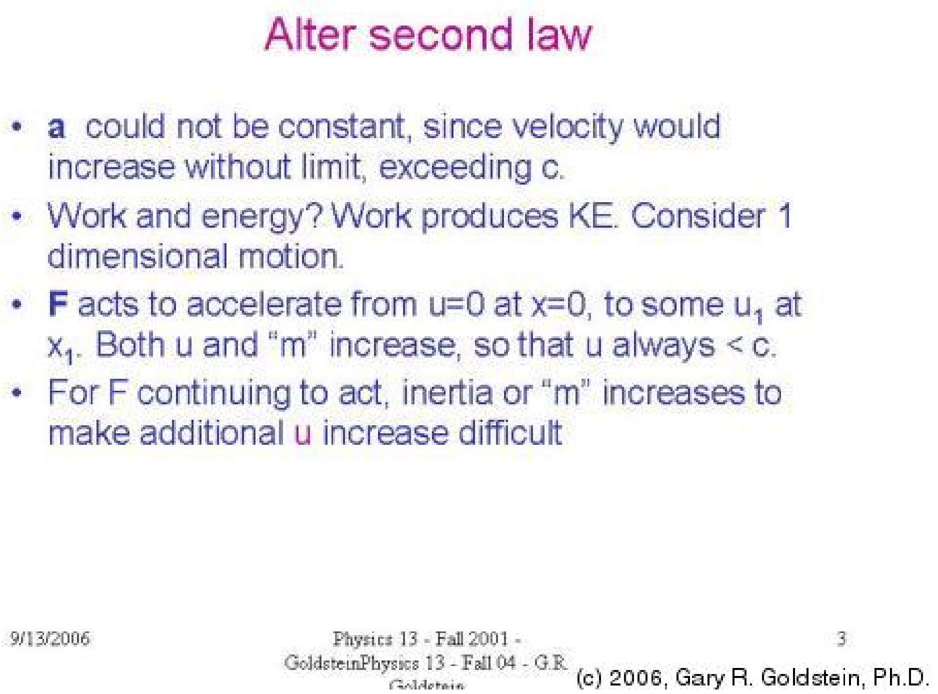
3. Alter second law



Alter second law

- **a** could not be constant, since velocity would increase without limit, exceeding c.
- Work and energy? Work produces KE. Consider 1 dimensional motion.
- **F** acts to accelerate from u=0 at x=0, to some u_1 at x_1. Both u and "m" increase, so that u always < c.
- For F continuing to act, inertia or "m" increases to make additional u increase difficult

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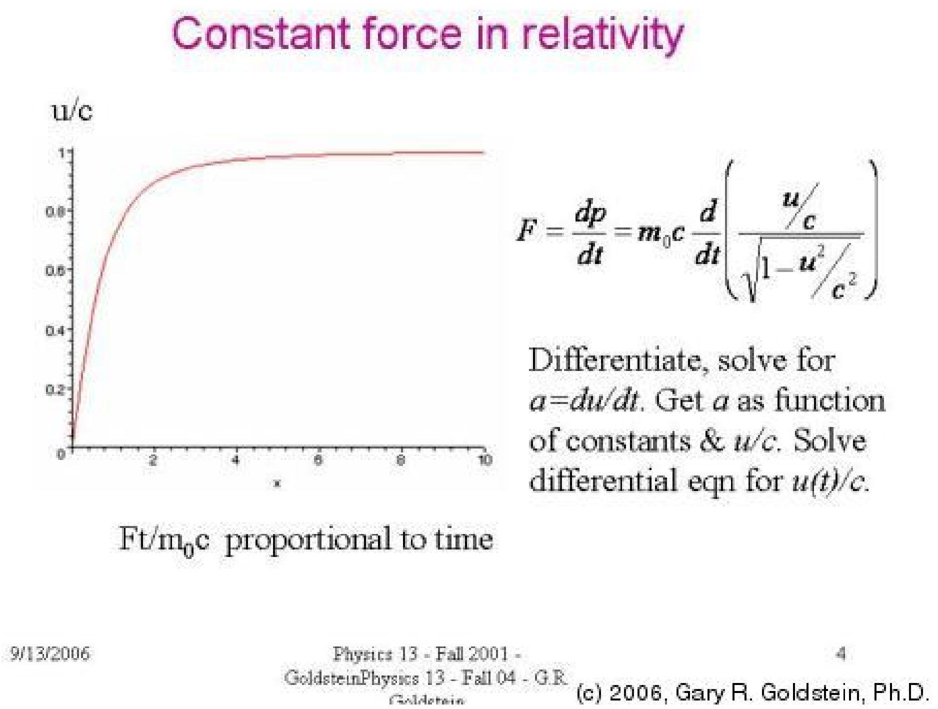
4. Constant force in relativity



Constant force in relativity

Ft/m_0c proportional to time

$$F=\frac{dp}{dt}=m_0c\,\frac{d}{dt}\left(\frac{u/c}{\sqrt{1-u^2/c^2}}\right)$$

Differentiate, solve for *a=du/dt*. Get *a* as function of constants & *u/c*. Solve differential eqn for *u(t)/c*.

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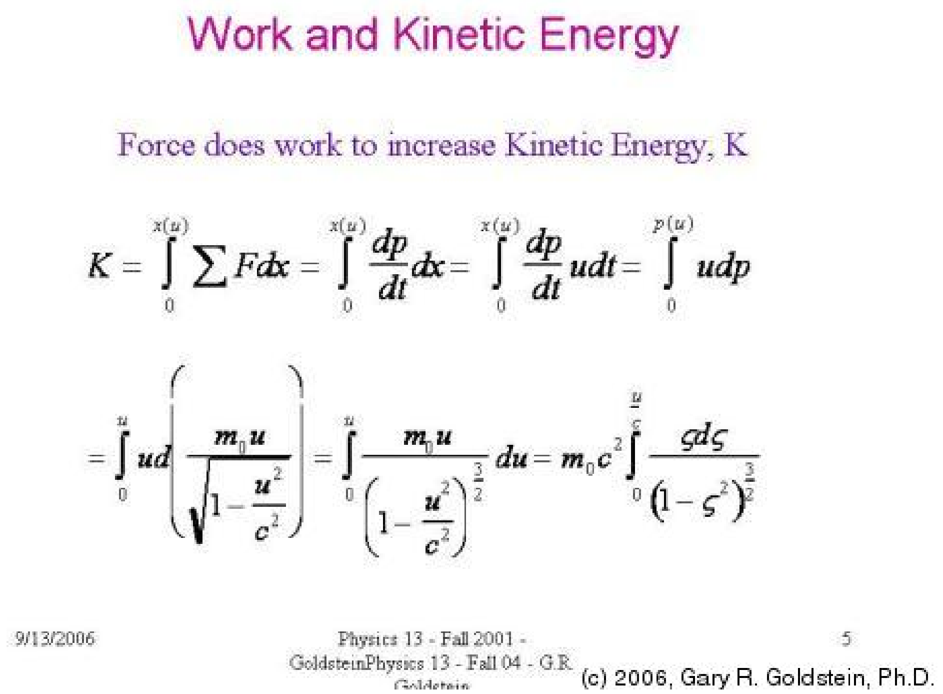
5. Work and Kinetic Energy



Work and Kinetic Energy

Force does work to increase Kinetic Energy, K

$$K = \int_0^{x(u)} \sum F dx = \int_0^{x(u)} \frac{dp}{dt} dx = \int_0^{x(u)} \frac{dp}{dt} u dt = \int_0^{p(u)} u dp$$

$$= \int_0^{u} u d\left(\frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = \int_0^{u} \frac{m_0 u}{\left(1-\frac{u^2}{c^2}\right)^{\frac{3}{2}}} du = m_0 c^2 \int_0^{\frac{u}{c}} \frac{\varsigma d\varsigma}{\left(1-\varsigma^2\right)^{\frac{3}{2}}}$$

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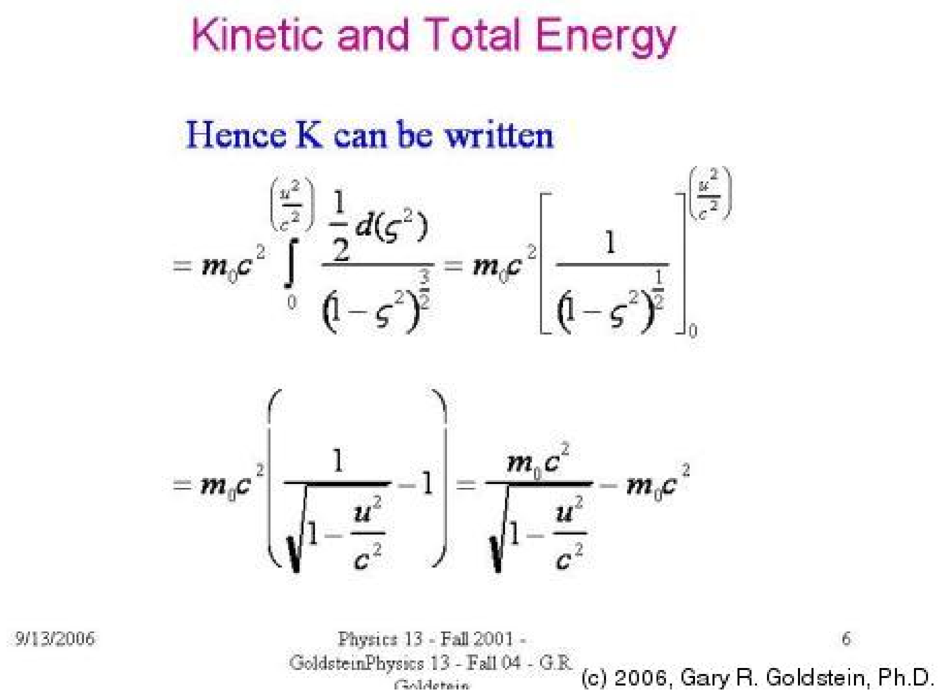
6. Kinetic and Total Energy



Kinetic and Total Energy

Hence K can be written

$$= m_0 c^2 \int_0^{\left(\frac{u^2}{c^2}\right)} \frac{\frac{1}{2} d(\varsigma^2)}{\left(1-\varsigma^2\right)^{\frac{3}{2}}} = m_0 c^2 \left[\frac{1}{\left(1-\varsigma^2\right)^{\frac{1}{2}}}\right]_0^{\left(\frac{u^2}{c^2}\right)}$$

$$= m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1\right) = \frac{m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}} - m_0 c^2$$

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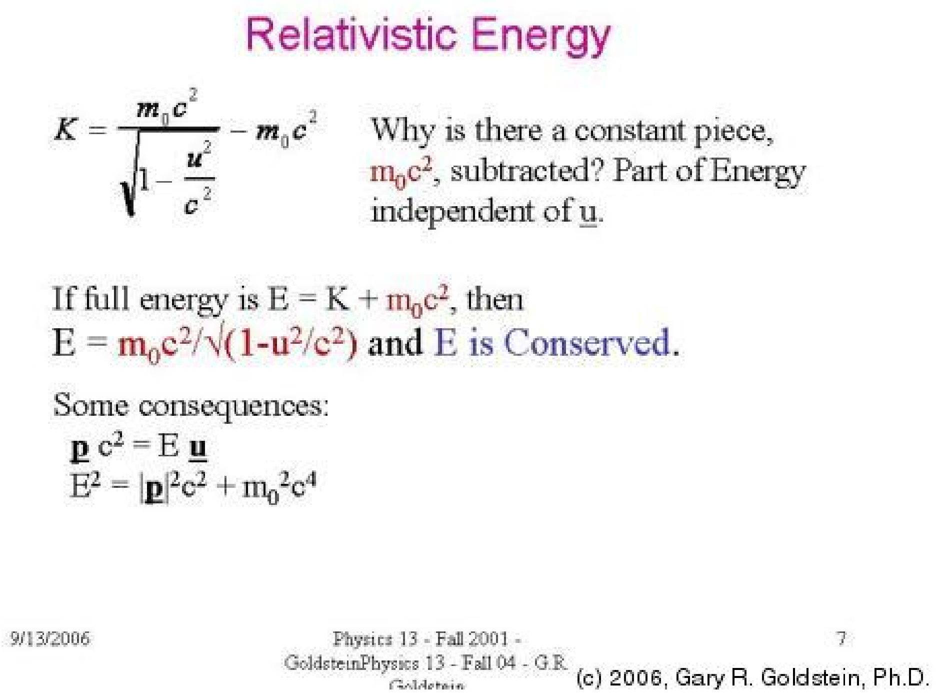
7. Relativistic Energy



Relativistic Energy

$$K = \frac{m_0 c^2}{\sqrt{1 - \frac{u^2}{c^2}}} - m_0 c^2$$

Why is there a constant piece, m_0c^2, subtracted? Part of Energy independent of u.

If full energy is $E = K + m_0c^2$, then
$E = m_0c^2/\sqrt{(1-u^2/c^2)}$ and E is Conserved.

Some consequences:
$\mathbf{p}\,c^2 = E\,\mathbf{u}$
$E^2 = |\mathbf{p}|^2c^2 + m_0^2c^4$

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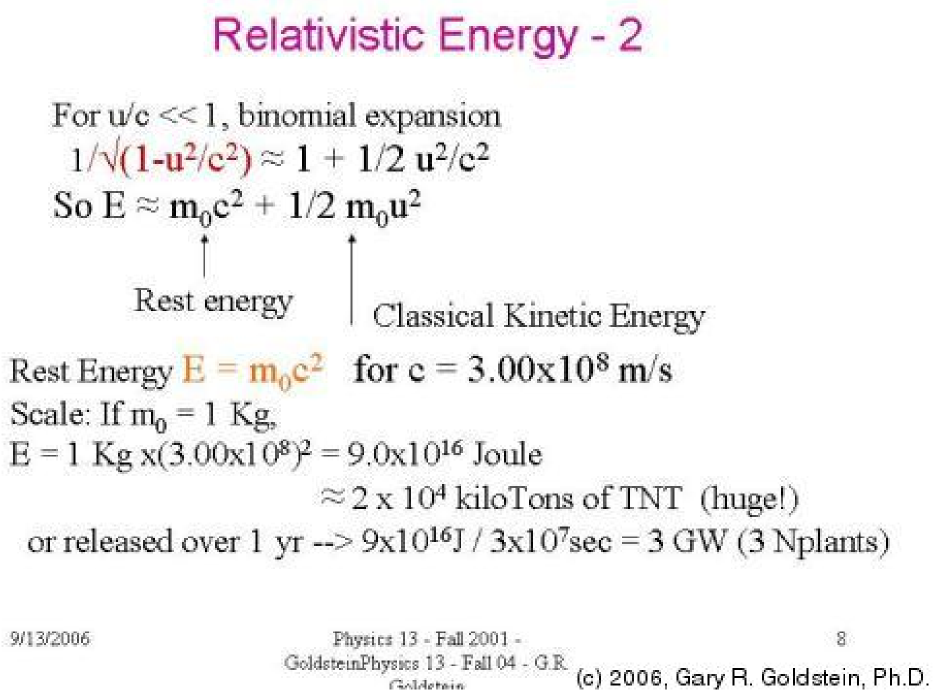
8. Relativistic Energy cont'd



Relativistic Energy - 2

For u/c << 1, binomial expansion
$1/\sqrt{(1-u^2/c^2)} \approx 1 + 1/2\, u^2/c^2$
So $E \approx m_0c^2 + 1/2\, m_0u^2$

Rest energy

Classical Kinetic Energy

Rest Energy $E = m_0c^2$ for $c = 3.00\text{x}10^8$ m/s
Scale: If $m_0 = 1$ Kg,
$E = 1\text{ Kg x}(3.00\text{x}10^8)^2 = 9.0\text{x}10^{16}$ Joule
$\approx 2\text{ x }10^4$ kiloTons of TNT (huge!)
or released over 1 yr --> $9\text{x}10^{16}\text{J} / 3\text{x}10^7\text{sec} = 3$ GW (3 Nplants)

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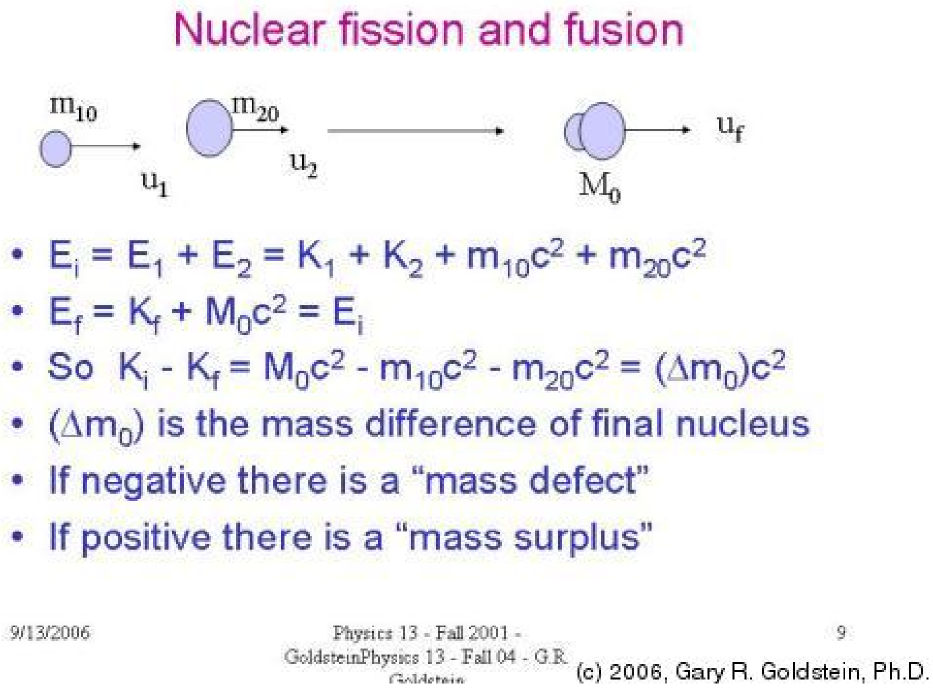
9. Nuclear fission and fusion



Nuclear fission and fusion

- $E_i = E_1 + E_2 = K_1 + K_2 + m_{10}c^2 + m_{20}c^2$
- $E_f = K_f + M_0c^2 = E_i$
- So $K_i - K_f = M_0c^2 - m_{10}c^2 - m_{20}c^2 = (\Delta m_0)c^2$
- (Δm_0) is the mass difference of final nucleus
- If negative there is a "mass defect"
- If positive there is a "mass surplus"

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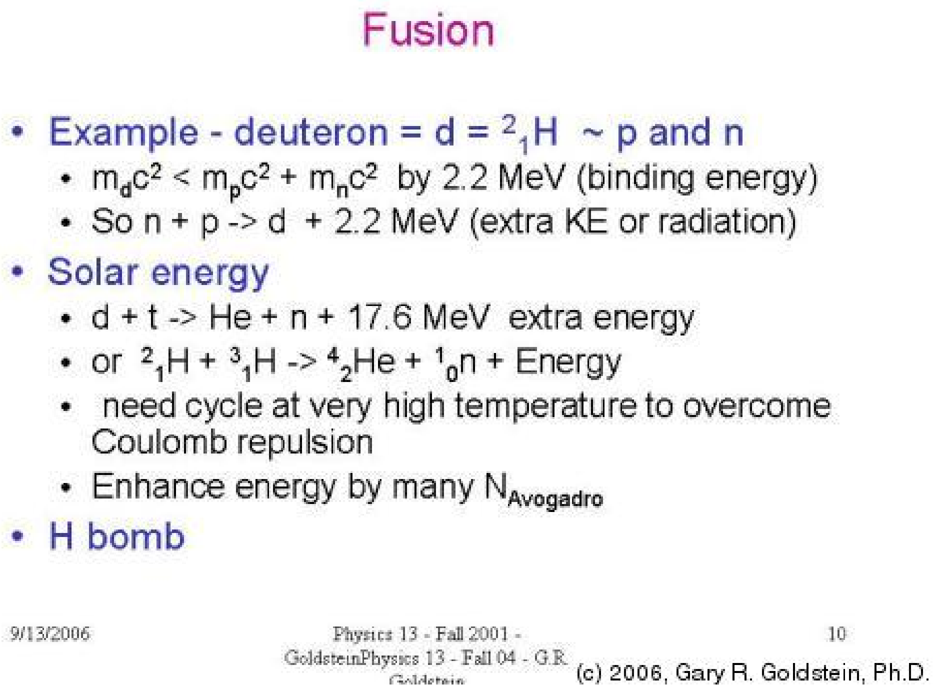
10. Fusion



Fusion

- Example - deuteron = d = $^2{}_1H$ ~ p and n
  - $m_dc^2 < m_pc^2 + m_nc^2$ by 2.2 MeV (binding energy)
  - So n + p -> d + 2.2 MeV (extra KE or radiation)
- Solar energy
  - d + t -> He + n + 17.6 MeV extra energy
  - or $^2{}_1H + {}^3{}_1H \rightarrow {}^4{}_2He + {}^1{}_0n$ + Energy
  - need cycle at very high temperature to overcome Coulomb repulsion
  - Enhance energy by many $N_{Avogadro}$
- H bomb

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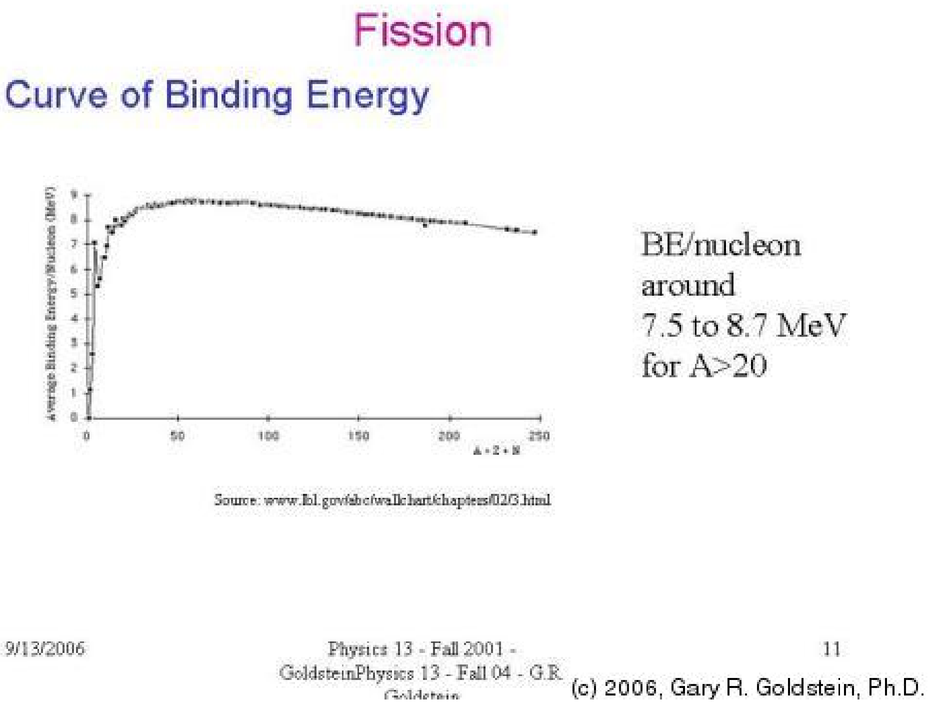
11. Fission



Fission

Curve of Binding Energy

BE/nucleon
around
7.5 to 8.7 MeV
for A>20

Source: www.lbl.gov/abc/wallchart/chapters/02/3.html

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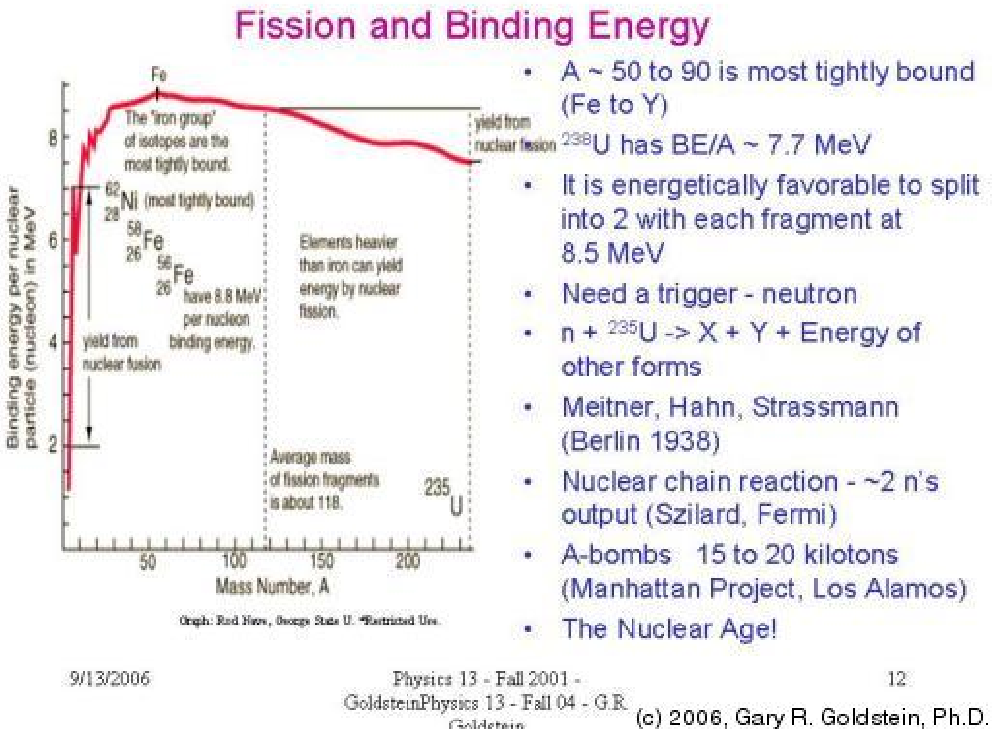
12. Lecture 5: Force, Energy, Fission: Slide 12



Fission and Binding Energy

Graph: Rod Nave, Georgia State U. *Restricted Use.

- A ~ 50 to 90 is most tightly bound (Fe to Y)
- ^{238}U has BE/A ~ 7.7 MeV
- It is energetically favorable to split into 2 with each fragment at 8.5 MeV
- Need a trigger - neutron
- n + ^{235}U -> X + Y + Energy of other forms
- Meitner, Hahn, Strassmann (Berlin 1938)
- Nuclear chain reaction - ~2 n's output (Szilard, Fermi)
- A-bombs 15 to 20 kilotons (Manhattan Project, Los Alamos)
- The Nuclear Age!

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