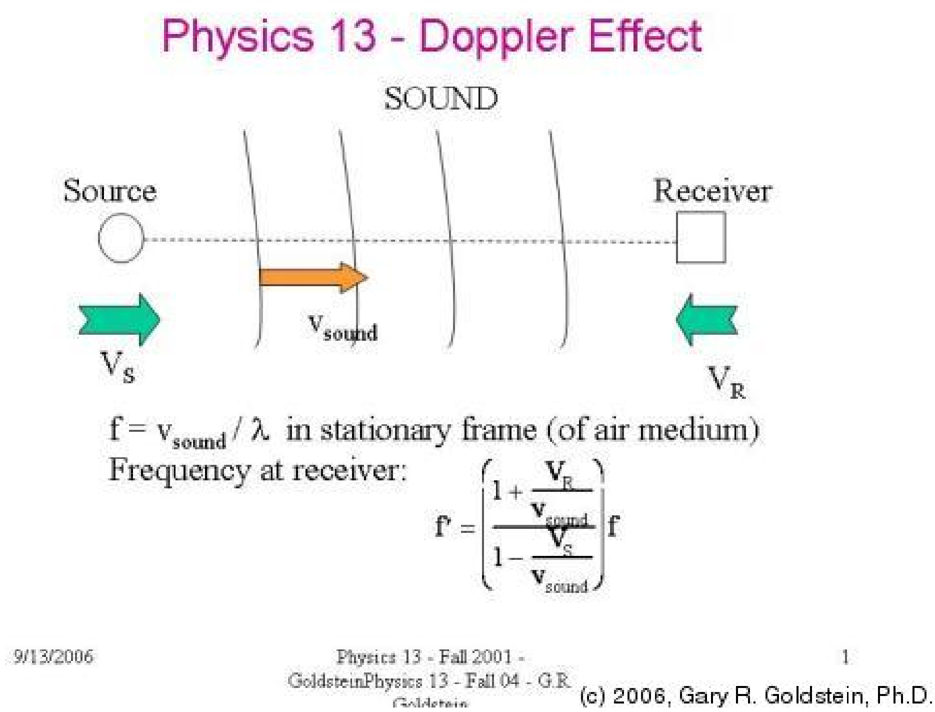
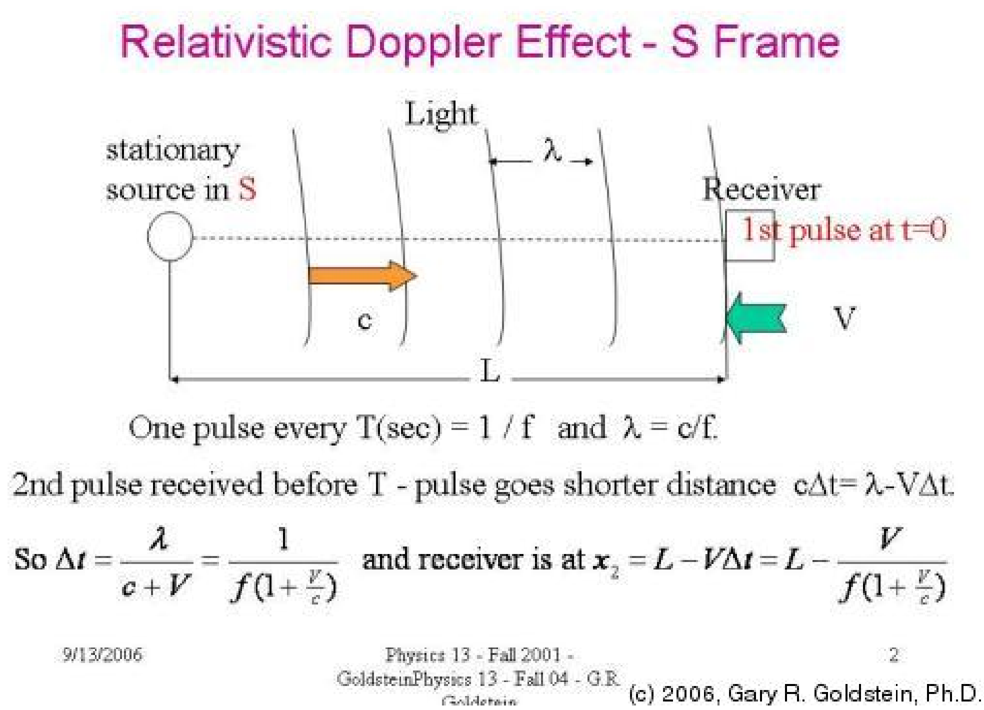
1. Physics 13 - Doppler Effect



2. Relativistic Doppler Effect - S Frame



3. Relativistic Doppler Effect - Receiver

Relativistic Doppler Effect - Receiver

- 2 events in S:
  - E_1: (L,0,0,0) and E_2: (x_2,0,0,Δt) = (L- V/[f(1+V/c)], 0, 0, 1/[f(1+V/c)])
- In S′ (V approaching source):
  - $t_1' = \gamma\,(t_1 + Vx_1/c^2) = \gamma\,(VL/c^2)$
  - $t_2' = \gamma\,(t_2 + Vx_2/c^2) = \gamma\,(\Delta t + V[L-V\Delta t]/c^2)$ $= t_1' + \gamma\,\Delta t\,(1-V^2/c^2) = t_1' + \Delta t\,/\,\gamma$
  - $\Delta t' = \Delta t\,/\,\gamma = 1\,/\,f' = 1\,/\,[\gamma\, f(1+V/c)]$
  - $f' = \gamma\, f(1+V/c) = [\sqrt{(1+V/c)}/\sqrt{(1-V/c)}]\, f$

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4. Relativistic Doppler Effect - Examples

Relativistic Doppler Effect - examples

$$f' = \sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}}\, f$$

1) Small V or V/c <<1, so $f' \approx (1 + V/c)\, f$ as in Classical case.
2) Edge of universe: e.g. V=c/2, $f'/f = \sqrt{(3/2)}/\sqrt{(1/2)} = \sqrt{3} = 1.7$ which is a huge shift. Quasars were discovered from this effect.

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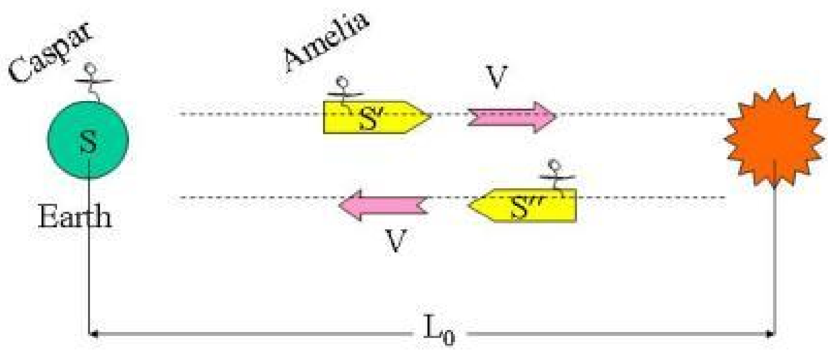
5. Twin Paradox

Twin Paradox (S)



L_0=12 lt yr = 12(years)c, V = 0.6 c,
so Δt=12c/0.6c=20 yr one way
and 20 yr return (quick turn around) according to Casper.
Casper ages 40 yr in time for Amelia's round trip. In **S**.

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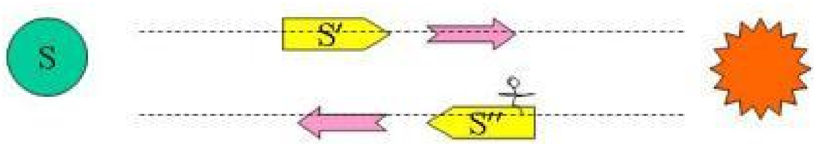
6. Twin Paradox cont'd

Twin Paradox (S′ and S″)



S′: $\Delta t' = \Delta t / \gamma = 20\times 4/5 = 16$ yr $\gamma = 1/\sqrt{(1-.6^2)} = 1/\sqrt{.64} = 1/.8 = 5/4$
Same for S″ so Amelia ages only 32 yr.
Note that $L' = L_0/\gamma = 12\times 4/5 = 9.6$ lt yr as Star approaches rocket at V=0.6c.

WHY doesn't Caspar age less as determined from Amelia's frame? Caspar stays in a *single* inertial frame, Amelia does *not*. They are not reciprocal.

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7. Twin Paradox and Communications 1

Twin Paradox and Communications 1

- Caspar (S) sends Amelia a radio birthday message every year, or $f_{Caspar}=1/yr=1yr^{-1}$.
- Amelia (S′) receives at
  - $f' = f\sqrt{(1-.6)}/\sqrt{(1+.6)} = 1/2\ yr^{-1}$ or 16/2=8 greetings in 16 yr of the outgoing trip
  - $f'' = f\sqrt{(1+.6)}/\sqrt{(1-.6)} = 2\ yr^{-1}$ or $16 \times 2=32$ greetings in 16 yr return trip (S′) for total of 40 Caspar birthdays

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8. Twin Paradox and Communications 2

Twin Paradox and Communications 2

- Amelia (S′ and S″) sends Caspar a radio birthday message every year, or $f_{Amelia}=1/yr=1yr^{-1}$.
- Caspar (S) receives at
  - $f = f'\sqrt{(1-.6)}/\sqrt{(1+.6)} = 1/2\ yr^{-1}$ for 20 yr + 12 yr (for last signal to reach him) or (20+12)/2=16 greetings for the outgoing trip
  - $f = f'\sqrt{(1+.6)}/\sqrt{(1-.6)} = 2\ yr^{-1}$ for $(20-12) \times 2=16$ greetings in return trip for total of 32 Amelia birthdays --- not reciprocal

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9. Velocity transformation

Velocity transformation

- S: object moving with velocity $\underline{u}$
- Use $u_x = dx/dt$, ... in S. In S'($\underline{V} = (V_x, 0, 0)$)?
  - $dx = \gamma\,(dx' + Vdt')$, $dy = dy'$, $dz = dz'$
  - $dt = \gamma\,(dt' + Vdx'/c^2)$ [$= \gamma dt'(1 + V\,u_x'/c^2)$]
  - So $u_x = dx/dt = (dx'/dt' + V)/(1 + [V/c^2]dx'/dt')$
  - or $u_x = (u_x' + V)/(1 + V\,u_x'/c^2)$
  - $u_y = dy/dt = dy'/[\gamma dt'(1 + [V/c^2]dx'/dt')]$
  - or $u_y = u_y'/[\gamma(1 + V\,u_x'/c^2)]$ and u_z similarly
- Example - light: $u_x = c \Rightarrow u_x' = c$

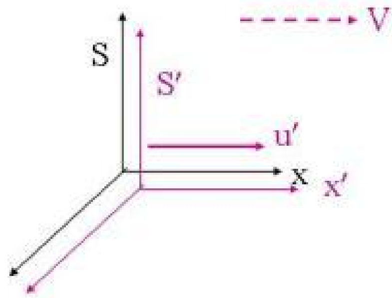
10. Velocity transformation- 1 dim

Velocity transformation- 1 dim

- In S': object moving with velocity $\underline{u}'$
  - S' moves at V relative to S
- Classically get $u = (u' + V)$
  - Would have c exceeded
- Relativity: $u = (u' + V)/(1 + Vu'/c^2)$
- Example - light: $u' = c \Rightarrow u = c$

11. Velocity transformation - example

Velocity transformation - example



Suppose S′ is frame of spaceship with V=0.8c. It launches a probe from its deck with u′=0.9c. What does Earth viewer see?

$u=(0.8c+0.9c)/(1+0.8\times0.9)=1.7c/1.72=0.99\,c$

- Can not exceed c, no matter how close u′ & V are to c.

12. Velocity transformation - examples

Velocity transformation - examples

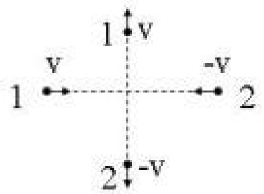
- 2 rockets approach earth from opposite directions along one axis with
- $u_1= 0.8c$ and $u_2= -0.8c$ from earth frame
- In rocket 1 frame $u_2'=(u_2-u_1)/(1+[-u_1]u_2/c^2)$ $= -1.6c/(1+.8^2)= -0.98c$
- Relative speed can never exceed 1

13. Momentum

Momentum

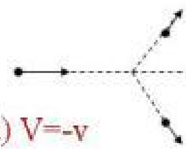
- Elastic collision in 2 frames

S



initial p_{xi}=0=mv-mv
p_{yi}=0
final p_{xf}=0
p_{yf}=0=mv-mv

S′ (2 initially at rest) V=-v



initial
$mv_{1xi}'=m(v_{1x}-V)/(1-Vv_{1x}/c^2)$
$=2mv/(1+v^2/c^2)=p_{1xi}'$
final
$mv_{1xf}'=m(-V)/(1)=mv=mv_{2xf}'$
$mv_{1yf}'=mv/\gamma=-mv_{2yf}'$

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14. Momentum problems

Momentum problems

- So initially in S′ total momentum is
  - $p_{xi}' = 2mv/(1+v^2/c^2)$ (with no y comp.)
- But finally
  - $p_{xf}' = 2mv$
- Momentum is not conserved in S′ !
- Need a relativistic definition of momentum

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15. Relativistic Momentum

Relativistic Momentum

- Ansatz: particle of velocity $\underline{\mathbf{v}}$ (space vector) and mass m has (vector) momentum
  - $\underline{\mathbf{p}} = m\,\underline{\mathbf{v}} / \sqrt{(1-v^2/c^2)}$ ($\underline{\mathbf{v}}$ is not frame velocity)
- Elastic example (S′):
  - $v_{1xi}' = (v_{1x}-V)/(1-Vv_{1x}/c^2) = 2v/(1+v^2/c^2)$ and $v_{1yi}' = 0$
  - So $p_{1xi}' = [2mv/(1+v^2/c^2)] / \sqrt{(1-(v_{1i}'/c)^2)}$

    some algebra required to yield
  - $= 2mv/(1-v^2/c^2) = p_{1xTOTAL}'$
  - And $p_{1xf}' = p_{2xf}' = mv / \sqrt{(1-(v_{1f}'/c)^2)}$

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16. Elastic example cont’d

Elastic example cont'd

- recall $v_{1yf}' = v/\gamma = -v_{2yf}'$
- so magnitude of $\underline{\mathbf{v}}_{1f}'$ is
- $v_{1f}' = \sqrt{[v^2+(v/\gamma)^2]} = \sqrt{[v^2+v^2(1-v^2/c^2)]} = v\sqrt{[2-v^2/c^2]}$
- and $1/\sqrt{[1-(v_{1f}'/c)^2]} = 1/\sqrt{\{1-(v^2[2-v^2/c^2]/c)^2\}}$
- $=1/(1-v^2/c^2)$
- Hence $p_{1xf}' = p_{2xf}' = mv/(1-v^2/c^2)$
- and total $p_{xi}' = p_{xf}'$ or $\underline{\mathbf{p}}_i' = \underline{\mathbf{p}}_f'$
- So conservation of relativistic vector momentum as required

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17. Momentum and mass

Momentum and mass

- Relativistic (3-) momentum
  - **p** = m **u** / √(1-u²/c²)   for (3-vector) velocity **u**
- is conserved in the absence of force **F**
- Note that **p** looks like classical momentum with "relativistic mass" m/ √(1-u²/c²)
- But only consider Rest Mass (u=0) m_0 and **p** = m_0 **u** / √(1-u²/c²)
- "Inertia" gets large as u->c

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