1. Physics 13 – Lengths, coordinates, and transformations...

Physics 13 – Lengths, coordinates, and transformations

Special Relativity

- Einstein (1905) required
  - Laws of Physics are independent of inertial frame *or* one can not detect absolute motion
  - Speed of light is independent of the motion of the source *or* c is velocity of EM wave in any inertial frame
- Time measurements are relative
- What about length measurements?
- Consider moving ruler of length L_0

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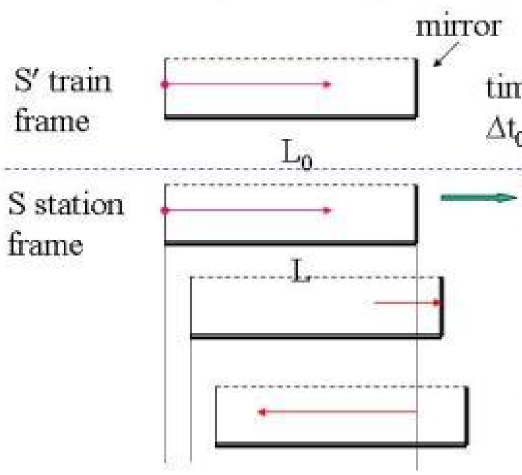
2. Measuring moving meter stick with light

Measuring moving meter stick with light



time round trip: 2×Length÷c
$\Delta t_0 = 2L_0/c$

time to mirror is longer:
$c\,\Delta t_1 = L + u\,\Delta t_1$ so
$\Delta t_1 = L/(c-u)$

time back to source is shorter:
$c\,\Delta t_2 = L - u\,\Delta t_2$ so
$\Delta t_2 = L/(c+u)$

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3. Meter stick length

Meter stick length

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2L}{c(1-\frac{u^2}{c^2})}$$

$$\text{But } \Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}} \qquad \text{time dilation}$$

$$\text{hence} \quad \frac{2L}{c(1-\frac{u^2}{c^2})} = \frac{2L_0}{c\sqrt{1-\frac{u^2}{c^2}}}$$

$$\text{or} \qquad L = L_0\sqrt{1-\frac{u^2}{c^2}} < L_0 (\text{proper length})$$

- Length Contraction
  - moving meter stick "appears" short
  - Note: to measure a moving stick must determine position of both ends **simultaneously**.

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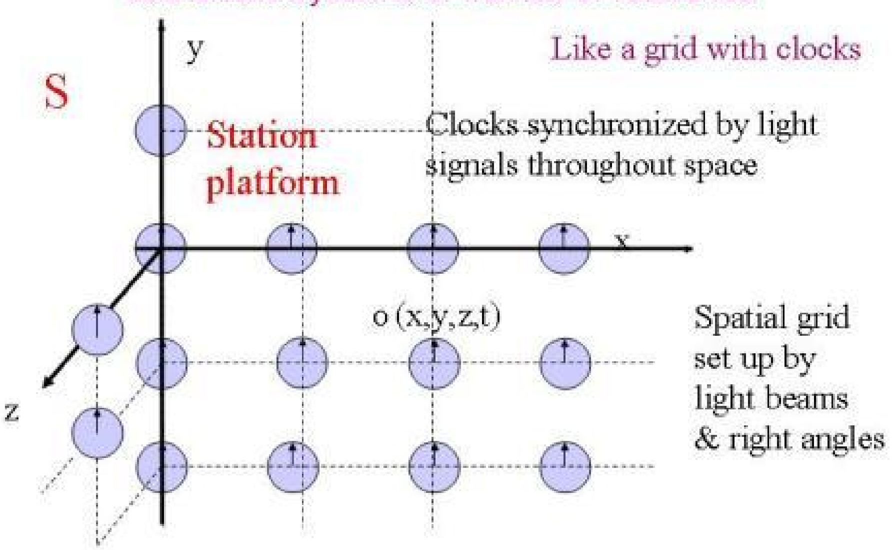
4. Setting up coordinate systems or frames of reference

All phenomena must be described by setting up coordinate systems or frames of reference



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5. Synchronizing clocks

Synchronizing clocks

- Clock 1 at (x,y,z)=(0,0,0) reads $t_1=0$ when it sends a signal to Clock 2 at (d,0,0).
- Clock 2 receives signal at $t_2=d/c$. So it must be set to read $t_2=d/c$ at that moment (not 0).
- When Clock 2 reads t_2 it sends a signal to Clock 3 at (2d,0,0). Latter gets signal at t_2+d/c and is set to display that value.
- Similar procedure is followed for the whole frame.

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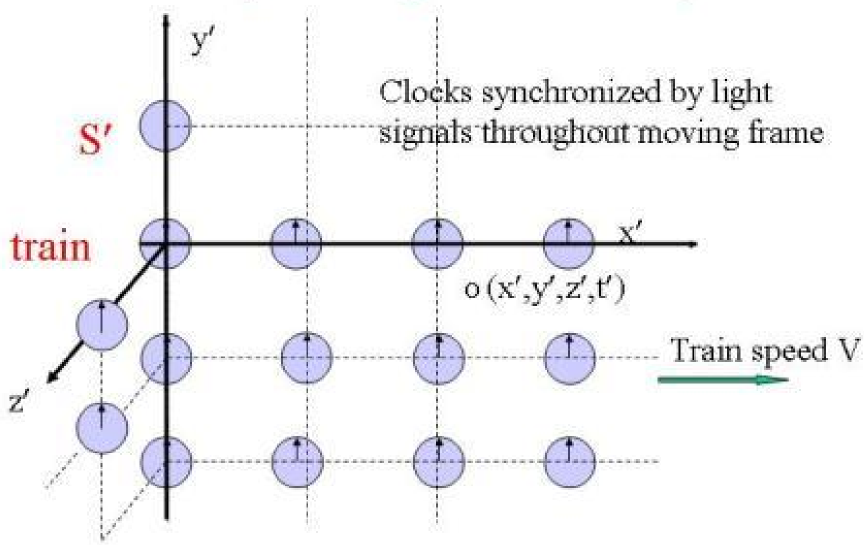
6. Lecture 3: Length, Coordinates, and Transformations: Slide...

Relatively moving coordinate system



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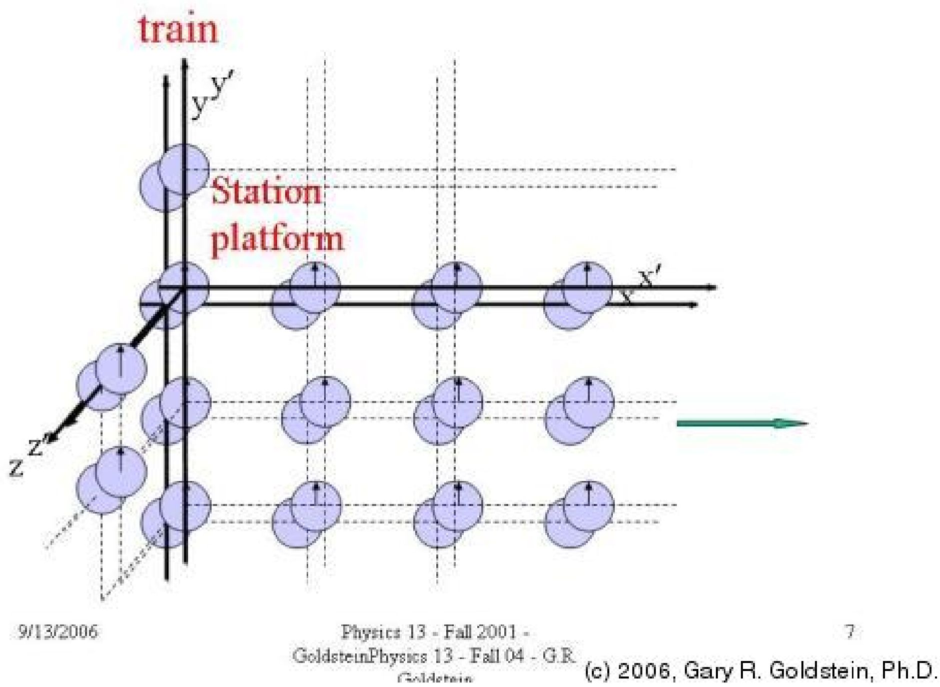
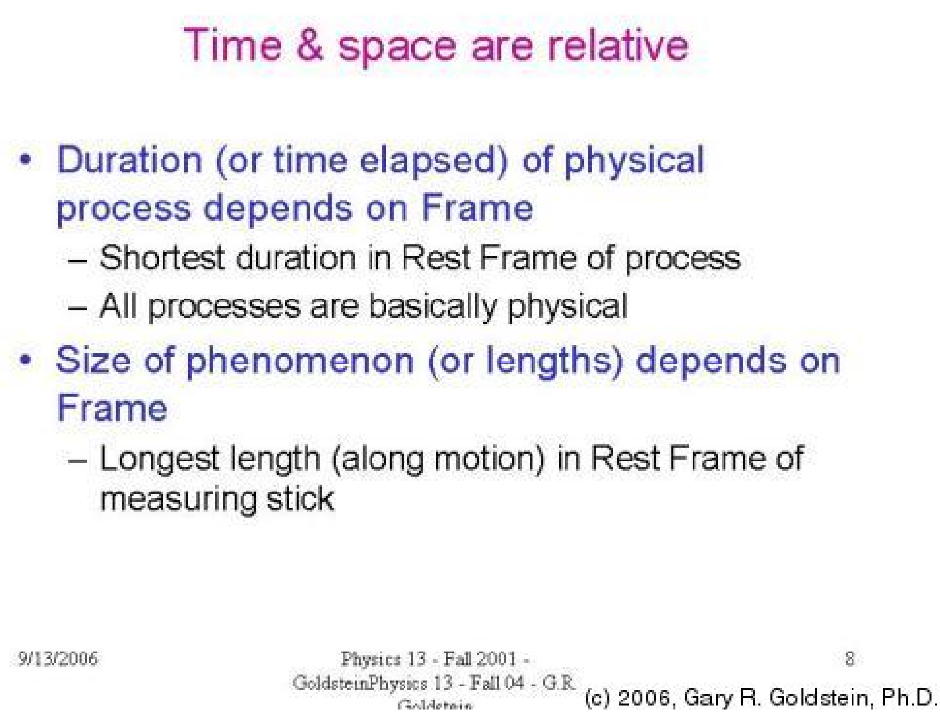
7. Lecture 3: Length, Coordinates, and Transformations: Slide...



8. Time & space are relative



9. Lorentz transformation

Lorentz transformation

- Let origins of both frames, S and S′, coincide at t=t′=0. Consider point (x,y,z,t) in S. In S′ :

  $x' = \gamma (x - Vt)$ $\gamma \equiv 1/\sqrt{(1-V^2/c^2)}$ (ubiquitous)

  $y' = y$

  $z' = z$

  $t' = \gamma (t - Vx/c^2)$ note signs

- Need x,y,z AND t to determine coordinates of space-time point in S′ - space-time continuum

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10. Inverse transformation

Inverse transformation

- S′ point or event $(x',y',z',t') \Rightarrow$ S (x,y,z,t)
  - $x = x'/\gamma + V(t'/\gamma + Vx/c^2) = V^2x/c^2 + (x' + Vt')/\gamma$
  - so $(1-V^2/c^2)x = (x' + Vt')/\gamma = x/\gamma^2$ or

  $x = \gamma (x' + Vt')$

  $y = y'$

  $z = z'$

  $t = \gamma (t' + Vx'/c^2)$ note + signs

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11. Example 1 of Lorentz Transformation

Example 1 of Lorentz Transformation

- Moving clock:
  - At rest S′: $(x', y', z', t') = (0,0,0,\Delta t')$ [after (0,0,0,0)]
  - In S: $x = \gamma\,(x' + Vt') = \gamma\, V\Delta t'$
  - and $t = \gamma\,(t' + Vx'/c^2) = \gamma\,\Delta t'$
  - or $\Delta t = \Delta t' / \sqrt{(1-u^2/c^2)}$ as before
  - Also note $x/t = \gamma V\Delta t' / \gamma\Delta t' = V$ as expected

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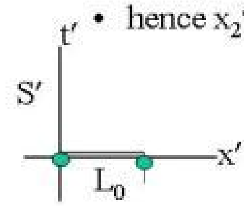
12. Example 2 of Lorentz Transformation

Example 2 of Lorentz Transformation

- Moving meter stick
  - In S′: ends of stick at x_1' and $x_2' = x_1' + L_0$ for all t'
  - In S: measure both ends at single time $t_1 = t_2$
    - but $x_1' = \gamma\,(x_1 - Vt_1)$
    - $x_2' = \gamma\,(x_2 - Vt_2) = \gamma\,(x_2 - Vt_1)$
    - hence $x_2' - x_1' = L_0 = \gamma\,(x_2 - x_1) = \gamma\, L$



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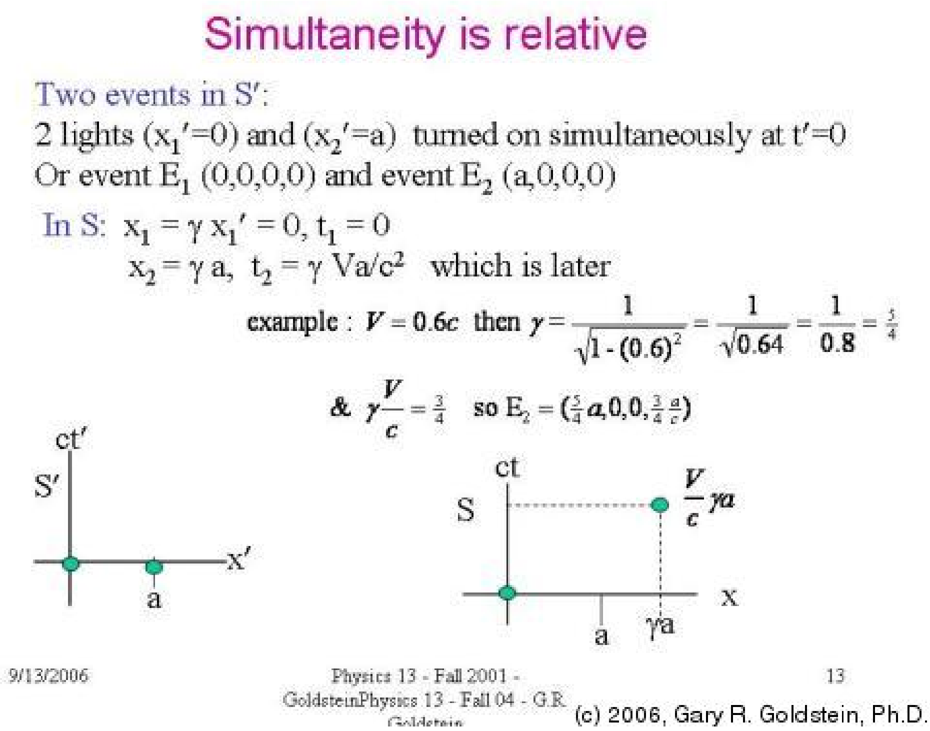
13. Simultaneity is relative



14. x-axis in S for different V’s