PHYSICS 6 SPRING 2006

Notes on the Twin Paradox
(to accompany slides, Lectures 6 and 7)

Gary R. Goldstein

From Caspar's frame of reference (fixed with respect to the earth) Amelia's clock (singular) on the trip outward would be slow compared to clocks (plural) in his frame. And the same conclusion holds on her return trip. So, indeed she would be younger than him when she is back on earth.

The apparent paradox is this: On her trip outward Caspar's clock (singular) will be slow compared to clocks (plural) in Amelia's frame. Similarly for her return trip. So how can it be that he is not younger than her?

The resolution of the paradox is that Amelia is first in one frame of reference (the outward trip) and then in a different frame of reference (for the return trip). So her time evolution is not in a single inertial frame and is thereby distinct from his.

Another way to see this is to consider what happens when Amelia's spaceship reverses direction. To switch from v=0.6c to v= - 0.6c requires a big acceleration. Her rockets have to provide a big reversing force. Assume this happens very quickly for her. For the short time that this acceleration occurs she is not at rest in an inertial frame, but in an accelerating frame. If she were able to see Caspar in that non-inertial time period, he would rapidly age. In that period she would also be experiencing a tremendous force pulling in the outbound direction. So her experience is one of being shifted from one frame to another, quite distinct from Caspar's.

From Slides:

S: (earth frame of reference)
Distance to star L_0=12 lt yr = 12 (years) $\times$ c
(= 12 $\times$[3.1x10^{7}sec] $\times$[3.0x10^{8}m/s]= 1.1x10^{17}m)
Speed of spaceship V = 0.6 c
so Δt=20 yr one way (12yr $\times$ c / 0.6c = 12/0.6 yr =20yr)
and 20 yr return (quick turn around). Casper ages 40 yr.

S′: (Spaceship outgoing)
$\Delta t' = \Delta t / \gamma$ =20$*$4/5 = 16 yr $\quad \gamma=1/\sqrt{(1-0.6^2)}$
$=1/\sqrt{.64}=1/.8=5/4$
Same for S″ (Spaceship returning), so Amelia ages only 32 yr.

Note that $L'=L_0/\gamma$ = 12$*$4/5=9.6 lt yr as Star approaches rocket at V=0.6c.

WHY doesn't Caspar age less as determined from Amelia's frame? Caspar stays in a *single* inertial frame, Amelia does *not*. They are not reciprocal.