Physics 6 (2006) Homework 4 Solutions

1 a) Prob for one 5 in 1st roll: (1/6)

b) Prob for 5 followed by anything other than 5 is $(1/6)\times(5/6)$
AND Prob for not-5 followed by 5 is $(5/6)\times(1/6)$. So total
Prob to get **ONLY** one 5 is sum of these two, or
$2\times(1/6)\times(5/6)=10/36=0.278$

c) Prob for at least one 5 will be the same as part (b) with the addition of the Prob for two 5's in two rolls. That latter is $(1/6)^2=1/36$.
The total is then 10/36 + 1/36 = 11/36 =0.306

d) In three rolls of the dice there can ONLY be 0 or 1 or 2 or 3 fives.
There are no other possibilities. So the addition of each of the possibilities must add to 1.0

e) Prob of ONLY one 5 in eight rolls? To get a five on the 1st roll and not the next 7 rolls is just Prob of $(1/6)\times(5/6)^7$. But there are seven other ways to get only one 5 in eight rolls. The final result will be

$8\times(1/6)\times(5/6)^7= 0.372$

f) There are many ways to have only two 5's turn up in eight rolls. How many?
7 ways for the 2nd 5 if the 1st roll produces a 5
+ 6 ways for the 3rd or 4th or … or 8th roll to be 5 if the 1st roll is not 5 but the 2nd roll is
+5+4+3+2+1 = 28 which is the same as $8!/(8-2)!\times(2)!$ From the general formula.
Each particular pair will have a Prob of $(1/6)^2\times(5/6)^6$. So the total Prob will be
$28\times(1/6)^2\times(5/6)^6=0.260$

2 a) Rate = 0.455 per atom/wk

b) ave. out of 35,000 is $35{,}000\times0.455$ = 15930 decay in one week.

c) After 1 week (35,000 – 15,930)=19070 remain. In next week there will be $19070\times0.455=8680$ decaying. That leaves 19070-8680=10,390 at the end of the 2nd week. In the last week $10{,}390\times0.455=4730$ will decay. That leaves roughly 10,390-4730=**5,660** left. (That is the same number of significant figures than we started with.)

You can use some algebra to do this. If N_0 is the initial number and r is the rate per atom per year, then in one year there will be $r\times N_0$ decaying. That means the number remaining in one year is $N_1 =(1-r)\times N_0$.
By the second year $r\times N_1$ decayed, which leaves $N_2 = (1-r)\times N_1 = (1-r)^2\times N_0$.
Then by three years there are $N_3 =(1-r)^3\times N_0$.
With r=0.455 (so 1-r = 0.545) and N_0 = 35,000 we get

$N_3 = (0.545)^3\times35{,}000 = 5666.$

For problem 3 I first did it with 3 place accuracy and then repeated it with 2 place accuracy. Either way will do for this problem.

3a) f = 89.7 MHz=$89.7\times10^{6}\,\text{sec}^{-1} = 8.97\times10^{7}s^{-1}$

$$f\lambda = c \quad \text{So } \lambda = \frac{c}{f} = \frac{3.0\times10^{8}\,m/s}{8.97\times10^{7}s^{-1}} = 0.334m$$

b) 1/1000 Watts = $10^{-3}J/s$

One photon has energy E=hf=$(6.63\times10^{-34}J\cdot s)\times 8.97\times10^{7}s^{-1}$

$$= 5.95\times10^{-26}J$$

So the number of photons needed to make up $10^{-3}J$ every second

will be $10^{-3}J \div (5.95\times10^{-26}J) = 1.68\times10^{22}$ photons in one second.

3.*a*) $f = 90.0\times10^{6}/\text{sec} = 9.00\times10^{7}/\text{sec}$

$$f\lambda = c \text{ (speed of light)} \quad \text{So } \lambda = c/f = \frac{3.00\times10^{8}\,m/\text{sec}}{9.00\times10^{7}/\text{sec}} = 3.33m$$

b) 1/1000 Watts = 1/1000 Joule/sec = $10^{-3}J/\text{sec}$

but one photon has energy $E = hf = 6.6\times10^{-34}J\,\text{sec}\times 9.00\times10^{7}/\text{sec}$

$$= 6.0\times10^{-26}J$$

So number of photons needed to make the $10^{-3}J$ in one second will be

$10^{-3}J \div 6.0\times10^{-26}J/photon = 1.7\times10^{22}$ photons

4.*a*) 510 nm is in the visible region (see the chart in Lecture 6 notes)

b) Wien's Law: peak of the blackbody spectrum, $\lambda_{peak} \propto 1/T$

So $\lambda_{peak1} \propto 1/T_1$ and $\lambda_{peak2} \propto 1/T_2$ with the same proportionality.

Then $\lambda_{peak1}/\lambda_{peak2} = (1/T_1)/(1/T_2) = T_2/T_1$

and $T_2 = T_1\times(\lambda_{peak1}/\lambda_{peak2}) = 5700K\times(5.10\times10^{-7}/1.06\times10^{-3}) = 2.74K$

5. For n = 3,2,1 E = -1.5eV,-3.4eV,-13.6eV.

So $3\rightarrow2$ releases (-1.5) - (-3.4) = 1.9eV photon

$3\rightarrow1$ releases (-1.5) - (-13.6) = 12.1eV photon

photon E = hf and $h = 6.6\times10^{-34}J\,\text{sec}\div1.6\times10^{-19}J/eV = 4.1\times10^{-15}eV\,\text{sec}$

Then $f(3\rightarrow2) = 1.9/4.1\times10^{-15} = 4.6\times10^{14}/\text{sec}$

$f(3\rightarrow1) = 12.1/4.1\times10^{-15} = 3.0\times10^{15}/\text{sec}$

Using $\lambda = c/f$ gives $6.5\times10^{-7}m$ and $10^{-7}m$ for both wavelengths

You can also include $2 \rightarrow 1$ since the electron that arrives at n=1 could have gone through 2 transitions, $3 \rightarrow 2$ and $2 \rightarrow 1$.

For $2 \rightarrow 1$ the energy released is (-3.4)-(-13.6)=10.2 eV photon.

Then $f(2 \rightarrow 1) = 10.2/ 4.1 \times 10^{-15} = 2.5 \times 10^{15}\ \text{sec}^{-1}$

and $\lambda = 3.0 \times 10^{8} / 2.5 \times 10^{15} = 1.2 \times 10^{-7} m$