

HOMEWORK 3 SOLUTIONS

1. The charged pion (π^+) decays has an average decay time of 2.6×10^{-8} sec.
 a. If a π^+ is travelling in a lab at $0.9999c$ what is its average decay time?

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{v^2}{c^2} = (0.9999)^2 = 0.9998, \quad 1 - \frac{v^2}{c^2} = 0.0002, \quad \sqrt{1 - \frac{v^2}{c^2}} = 0.01414,$$

$$\text{so } 1/\sqrt{1 - \frac{v^2}{c^2}} = 71 \quad \text{and } t = 71 \times 2.6 \times 10^{-8} = 1.8 \times 10^{-6} \text{ sec}$$

- b. How far does it travel before decaying (on the average)?
 $d = vt = 3 \times 10^8 \times 1.8 \times 10^{-6} = 5.4 \times 10^2 \text{ m}$
- c. The π^+ turns into three particles (usually muon and 2 neutrinos) when it decays. Suppose the muon is produced along the direction of the π^+ motion and has a velocity of $0.99c$ in the π^+ rest frame. What is the muon velocity in the lab?

In π^+ rest frame (S') muon has $u' = 0.99c$, but π^+ is moving at $V = 0.9999c$ in lab frame (S). Then

$$u = \frac{u' + V}{1 + \frac{Vu'}{c^2}} = \frac{0.99c + 0.9999c}{1 + (0.99 \times 0.9999)} = \frac{1.9899c}{1.989901c} = 0.9999995c$$

2. Judy and Andy are twins. Judy gets on a rocket that goes to a star system 20 light years away from earth (as measured from earth, i.e. Andy's frame of reference).
- a. She travels at $0.8c$. How many years does it take her to reach the star according to Andy?
 $t = 20 \text{ c yr} / 0.8 \text{ c} = 25 \text{ yr}$
- b. What will Judy's clock read when she reaches the star?
 Judy's clock runs slow according to Andy, via
 $t(\text{Judy}) = 25 \times \sqrt{(1 - 0.8^2)} = 25 \times \sqrt{(1 - 0.64)}$
 $= 25 \times \sqrt{(0.36)} = 25 \times 0.6 = 15 \text{ yr}$
- c. What distance will Judy have measured?
 $d = vt = 0.8 \text{ c} \times 15 \text{ yr} = 12 \text{ lt yr}$
or $d(\text{Judy}) = 20 \text{ lt yr} \times \sqrt{(1 - 0.8^2)} = 12 \text{ lt yr}$
- d. Judy sends a radio message to Andy when she reaches the star announcing her clock reading at that time. The signal will take 20 yrs to get back to Andy (traveling at the speed of light), so he will receive the signal showing that Judy is 15 years older, but at **$25 \text{ yr} + 20 \text{ yr} = 45 \text{ yr}$ after Judy left.**

3. In nuclear fission of Plutonium239 about 1/2000 of the mass-energy is converted to radiation and kinetic energy of the fission fragments.
- What is the gram molecular weight of Pu239? **239 grams = 0.239 Kg**. How many Pu239 nuclei are in 10 moles (or in the equivalent of 10 gram molecular weights)? $10 \times 6.0 \times 10^{23} = 6.0 \times 10^{24}$ nuclei.
 - What is the mass of one Pu239 nucleus?
 $0.239 \text{ Kg} / 6.0 \times 10^{23} = 3.9 \times 10^{-25} \text{ Kg} = m_0$
 What is the rest energy? $m_0 c^2 = 4.0 \times 10^{-25} (3.0 \times 10^8)^2$
 $= 3.6 \times 10^{-8} \text{ J}$
 - For one such fission how much energy is converted in units of Joules?
 $3.6 \times 10^{-8} \text{ J} / 2000 = 1.8 \times 10^{-11} \text{ J}$
 - When 10 moles undergo fission how much energy is converted? $1.8 \times 10^{-11} \text{ J} \times 6.0 \times 10^{24} = 1.0 \times 10^{14} \text{ J}$
4. Estimate the amount of electricity or electrical power that is consumed in the greater Boston area. How much mass would have to be converted to energy in order to supply that electrical power?
A typical electric bill for a family is the order of \$100 per month. At \$0.10 per KiloWatt hour that is 1000 KWh. (As usual with these kinds of estimations, order of magnitude is what is aimed for, not precision.)

In one month there are 30 days \times 24 hours = 720 hours. That means that the equivalent of 1000/720 KiloWatts = 1.4 KW of electrical power, or the burning of fourteen 100 watt light bulbs burning all the time.

If 0.5 million families are using that electrical power, that means $1.4 \text{ KW} \times 0.5 \times 10^6 = 0.7 \times 10^9 \text{ W} = 0.7 \text{ GigaWatts}$
 or $0.7 \times 10^9 \text{ J/s}$. In terms of rest energy per second, that is $0.7 \times 10^9 \text{ J}$ or $0.7 \times 10^9 \text{ J/c}^2 = 7.8 \times 10^{-9} \text{ Kg}$ of mass per second.

Over a whole year ($3.2 \times 10^7 \text{ sec}$) that is only $7.8 \times 10^{-9} \times 3.2 \times 10^7 = 0.25 \text{ Kg}$ (or about half a pound). From problem 3, we see that that amount of mass is about one mole of Pu239.