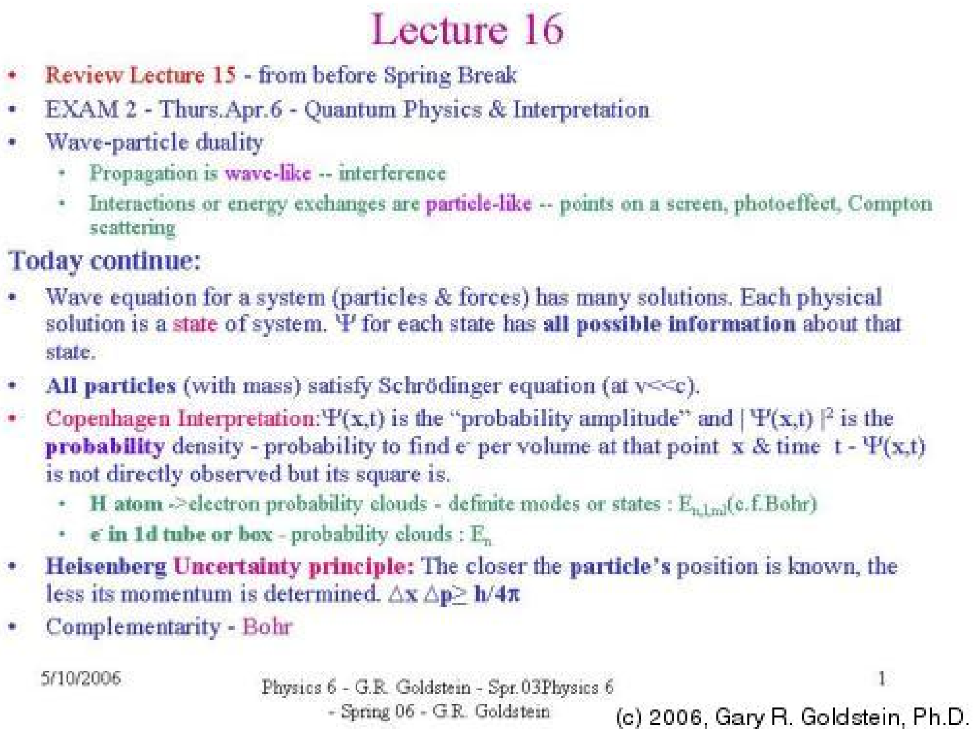
1. Lecture 16



Lecture 16

- **Review Lecture 15** - from before Spring Break
- EXAM 2 - Thurs.Apr.6 - Quantum Physics & Interpretation
- Wave-particle duality
  - Propagation is **wave-like** -- interference
  - Interactions or energy exchanges are **particle-like** -- points on a screen, photoeffect, Compton scattering

Today continue:

- Wave equation for a system (particles & forces) has many solutions. Each physical solution is a state of system. Ψ for each state has **all possible information** about that state.
- **All particles** (with mass) satisfy Schrödinger equation (at v<<c).
- Copenhagen Interpretation: $\Psi(x,t)$ is the "probability amplitude" and $|\Psi(x,t)|^2$ is the **probability** density - probability to find e^- per volume at that point **x** & time t - $\Psi(x,t)$ is not directly observed but its square is.
  - **H atom** ->electron probability clouds - definite modes or states : $E_{n,l,ml}$(c.f.Bohr)
  - **e^- in 1d tube or box** - probability clouds : E_n
- **Heisenberg Uncertainty principle:** The closer the **particle's** position is known, the less its momentum is determined. $\Delta x\ \Delta p \geq h/4\pi$
- Complementarity - Bohr

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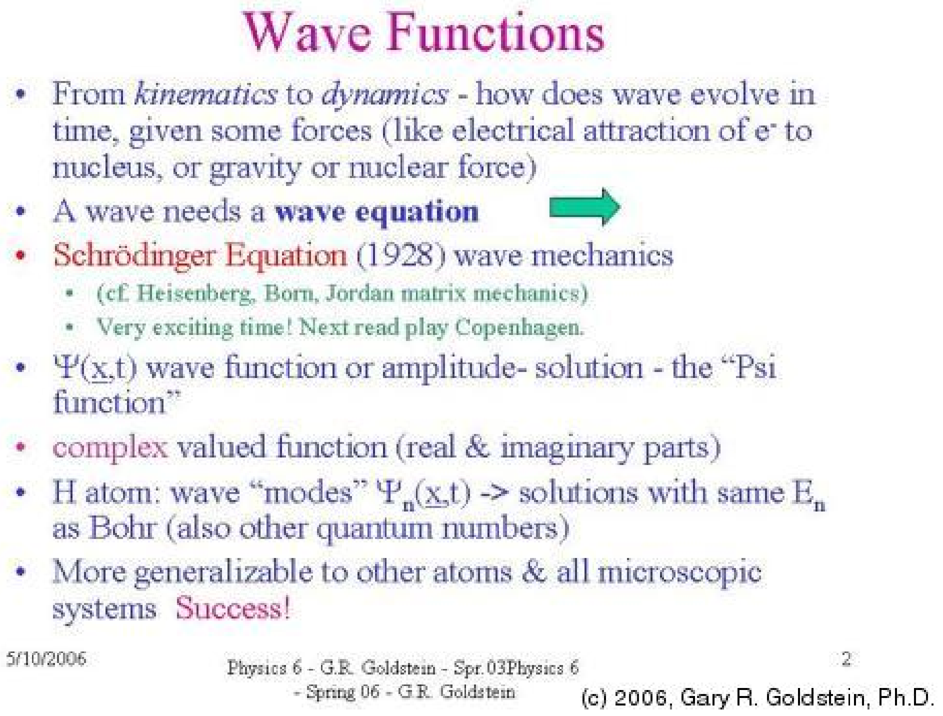
2. Wave Functions



Wave Functions

- From *kinematics* to *dynamics* - how does wave evolve in time, given some forces (like electrical attraction of e^- to nucleus, or gravity or nuclear force)
- A wave needs a **wave equation** ➡
- Schrödinger Equation (1928) wave mechanics
  - (cf. Heisenberg, Born, Jordan matrix mechanics)
  - Very exciting time! Next read play Copenhagen.
- $\Psi(\underline{x},t)$ wave function or amplitude- solution - the "Psi function"
- complex valued function (real & imaginary parts)
- H atom: wave "modes" $\Psi_n(\underline{x},t)$ -> solutions with same E_n as Bohr (also other quantum numbers)
- More generalizable to other atoms & all microscopic systems Success!

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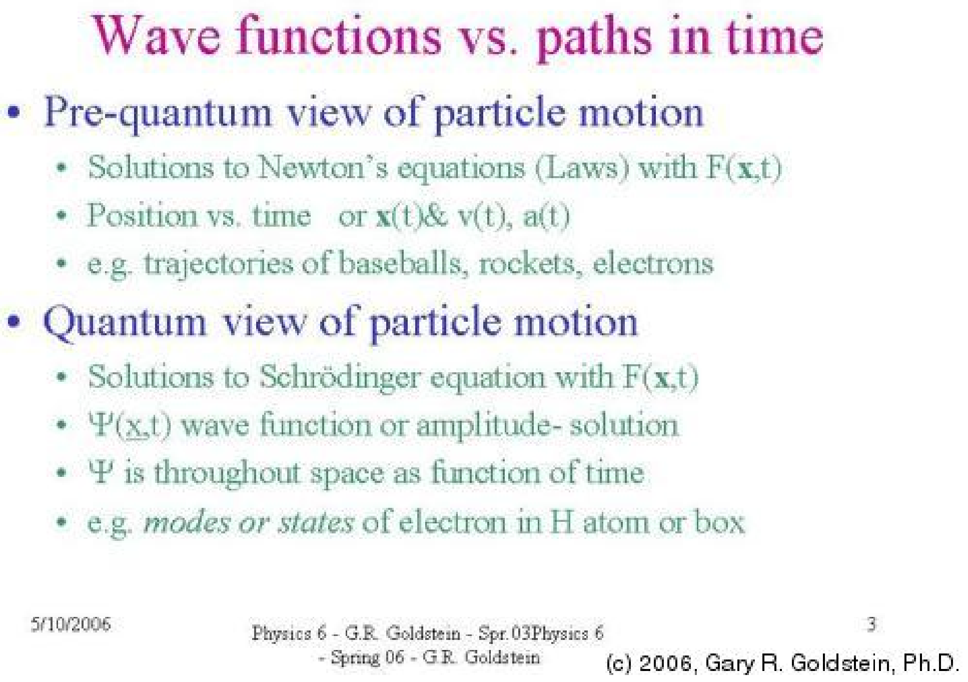
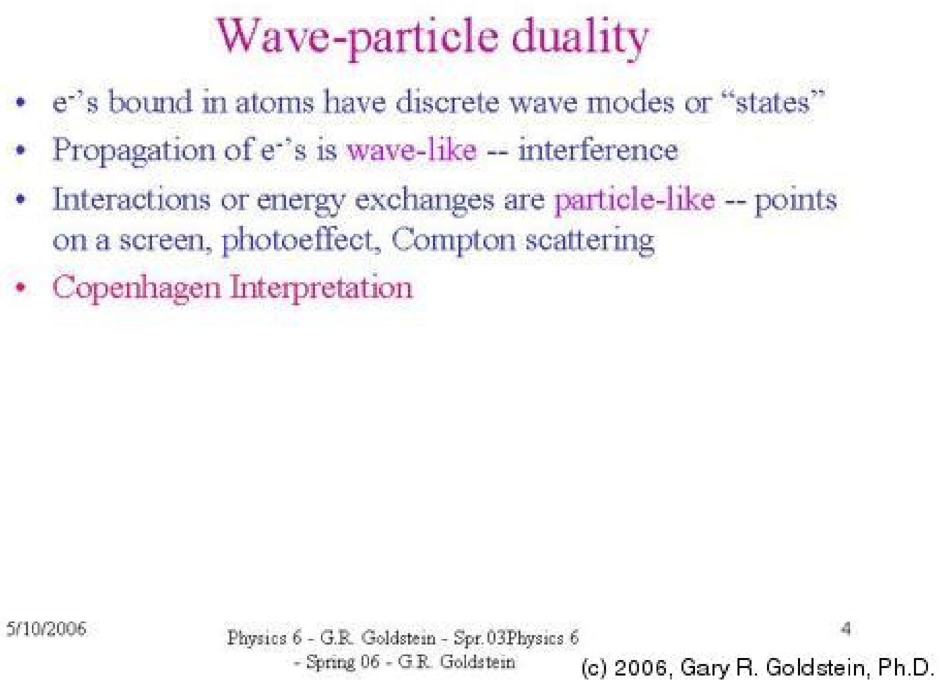
3. Wave functions vs. paths in time



Wave functions vs. paths in time

- Pre-quantum view of particle motion
  - Solutions to Newton's equations (Laws) with $F(\mathbf{x},t)$
  - Position vs. time or $\mathbf{x}(t)$& $v(t)$, $a(t)$
  - e.g. trajectories of baseballs, rockets, electrons
- Quantum view of particle motion
  - Solutions to Schrödinger equation with $F(\mathbf{x},t)$
  - $\Psi(\underline{x},t)$ wave function or amplitude- solution
  - Ψ is throughout space as function of time
  - e.g. *modes or states* of electron in H atom or box

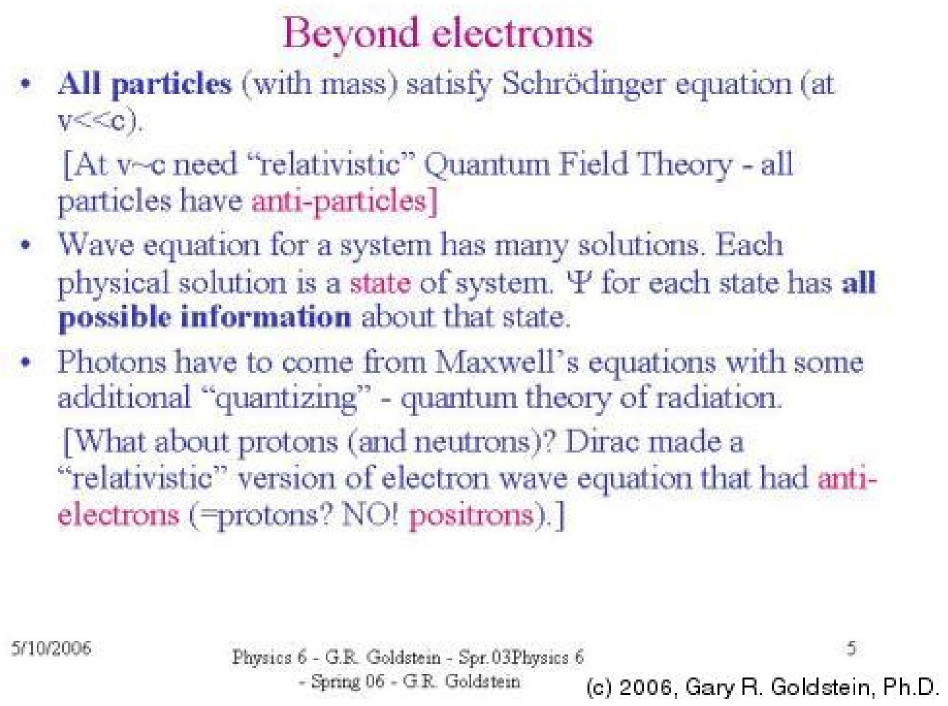
4. Wave-particle duality



Wave-particle duality

- e^-'s bound in atoms have discrete wave modes or "states"
- Propagation of e^-'s is wave-like -- interference
- Interactions or energy exchanges are particle-like -- points on a screen, photoeffect, Compton scattering
- Copenhagen Interpretation

5. Beyond electrons



Beyond electrons

- **All particles** (with mass) satisfy Schrödinger equation (at v<<c).

  [At v~c need "relativistic" Quantum Field Theory - all particles have anti-particles]
- Wave equation for a system has many solutions. Each physical solution is a state of system. Ψ for each state has **all possible information** about that state.
- Photons have to come from Maxwell's equations with some additional "quantizing" - quantum theory of radiation.

  [What about protons (and neutrons)? Dirac made a "relativistic" version of electron wave equation that had anti-electrons (=protons? NO! positrons).]

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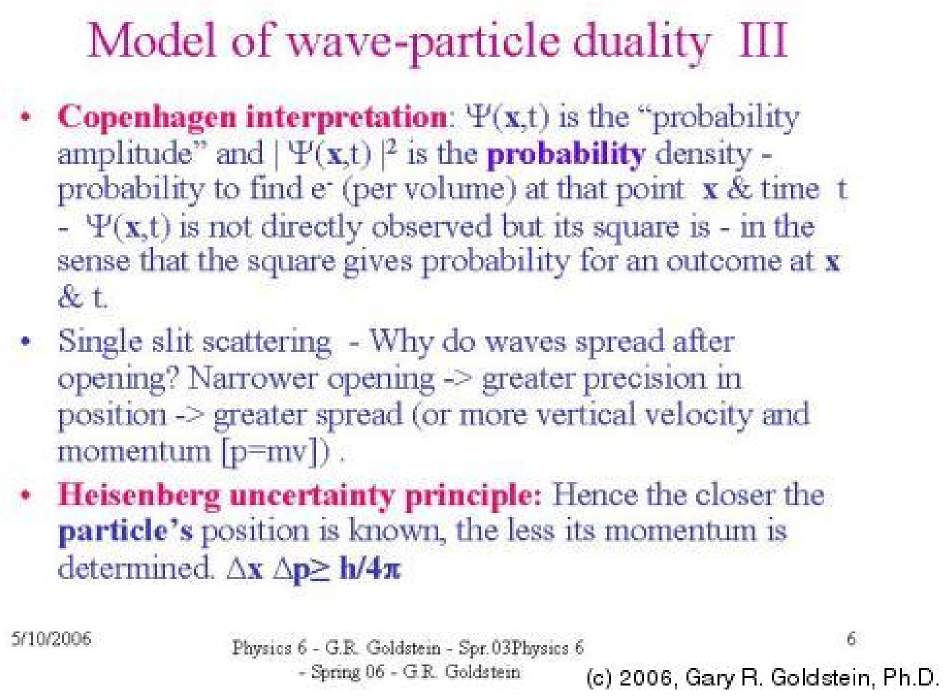
6. Model of wave-particle duality III



Model of wave-particle duality III

- **Copenhagen interpretation**: $\Psi(\mathbf{x},t)$ is the "probability amplitude" and $|\Psi(\mathbf{x},t)|^2$ is the **probability** density - probability to find e^- (per volume) at that point $\mathbf{x}$ & time t - $\Psi(\mathbf{x},t)$ is not directly observed but its square is - in the sense that the square gives probability for an outcome at $\mathbf{x}$ & t.
- Single slit scattering - Why do waves spread after opening? Narrower opening -> greater precision in position -> greater spread (or more vertical velocity and momentum [p=mv]) .
- **Heisenberg uncertainty principle:** Hence the closer the **particle's** position is known, the less its momentum is determined. $\Delta \mathbf{x}\ \Delta \mathbf{p} \geq \mathbf{h/4\pi}$

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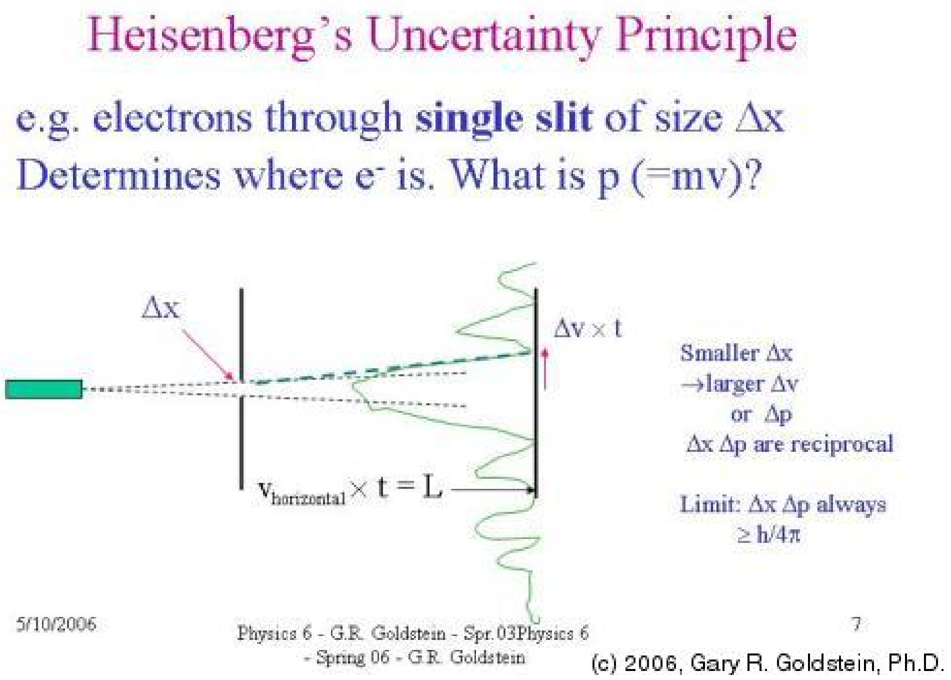
7.

Heisenberg’s Uncertainty Principle



Heisenberg’s Uncertainty Principle

e.g. electrons through **single slit** of size Δx
Determines where e^- is. What is p (=mv)?

Smaller Δx
$\rightarrow$larger Δv
or Δp
$\Delta x\ \Delta p$ are reciprocal

Limit: $\Delta x\ \Delta p$ always
$\geq h/4\pi$

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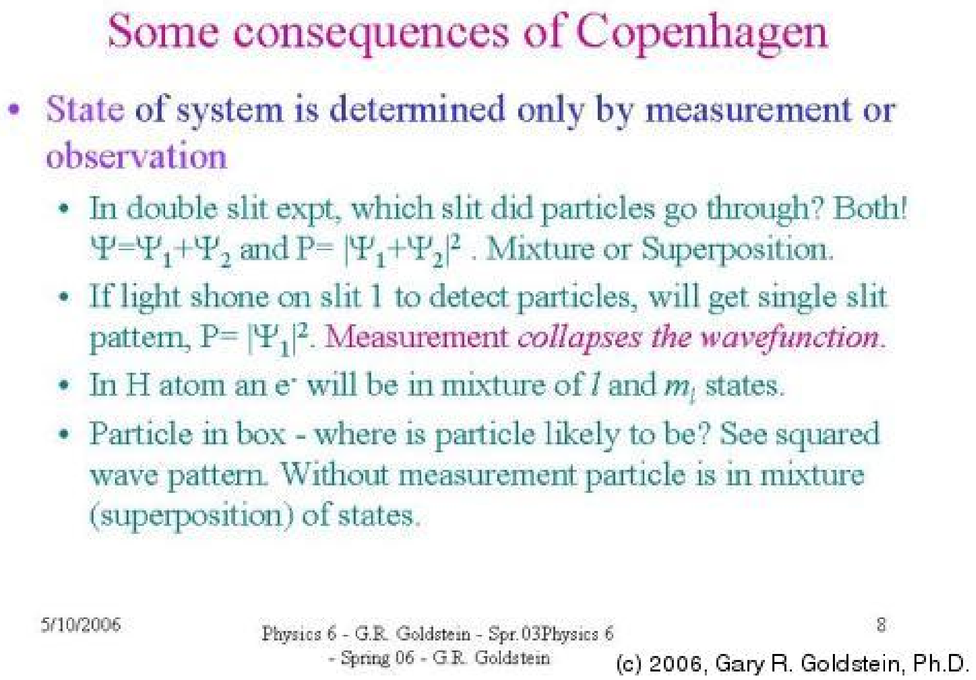
8.

Some consequences of Copenhagen



Some consequences of Copenhagen

- State of system is determined only by measurement or observation
  - In double slit expt, which slit did particles go through? Both! $\Psi=\Psi_1+\Psi_2$ and P= $|\Psi_1+\Psi_2|^2$. Mixture or Superposition.
  - If light shone on slit 1 to detect particles, will get single slit pattern, P= $|\Psi_1|^2$. Measurement *collapses the wavefunction.*
  - In H atom an e^- will be in mixture of l and m_l states.
  - Particle in box - where is particle likely to be? See squared wave pattern. Without measurement particle is in mixture (superposition) of states.

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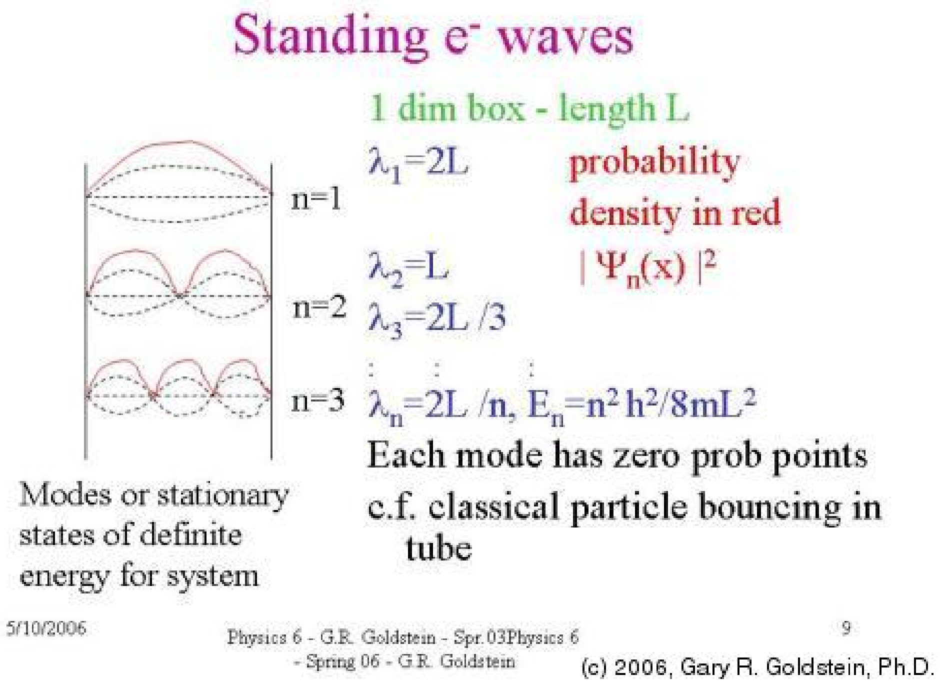
9. Standing e- waves



Standing e^- waves

1 dim box - length L

$\lambda_1=2L$ probability density in red $|\Psi_n(x)|^2$

$\lambda_2=L$

$\lambda_3=2L/3$

: : :

$\lambda_n=2L/n$, $E_n=n^2h^2/8mL^2$

Each mode has zero prob points

c.f. classical particle bouncing in tube

Modes or stationary states of definite energy for system

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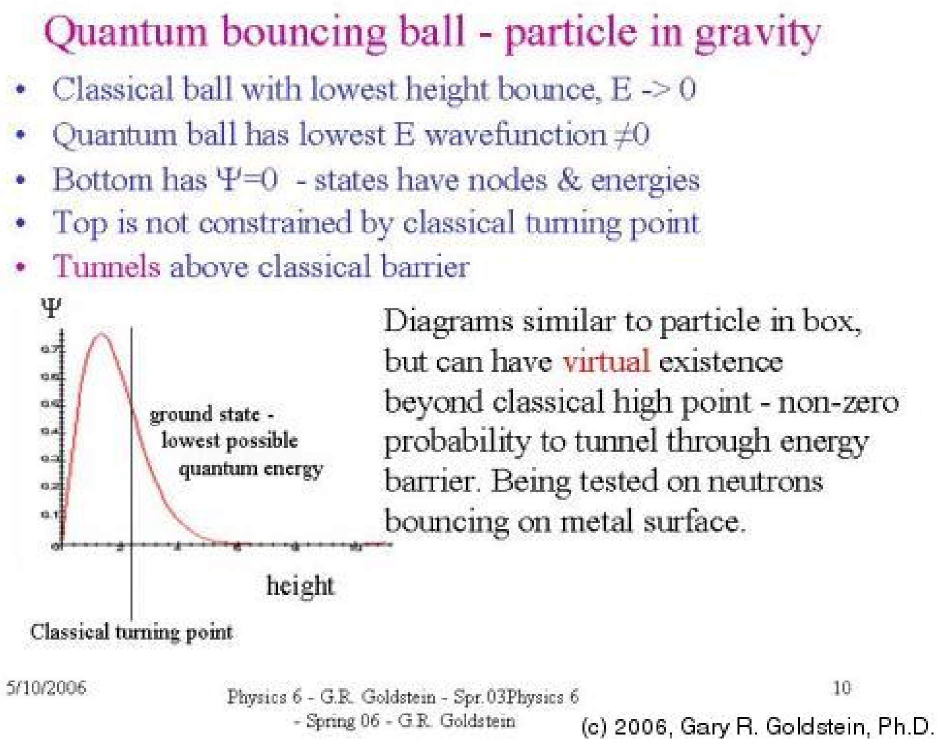
10. Quantum bouncing ball - particle in gravity



Quantum bouncing ball - particle in gravity

- Classical ball with lowest height bounce, E -> 0
- Quantum ball has lowest E wavefunction $\neq 0$
- Bottom has $\Psi=0$ - states have nodes & energies
- Top is not constrained by classical turning point
- Tunnels above classical barrier

Diagrams similar to particle in box, but can have virtual existence beyond classical high point - non-zero probability to tunnel through energy barrier. Being tested on neutrons bouncing on metal surface.

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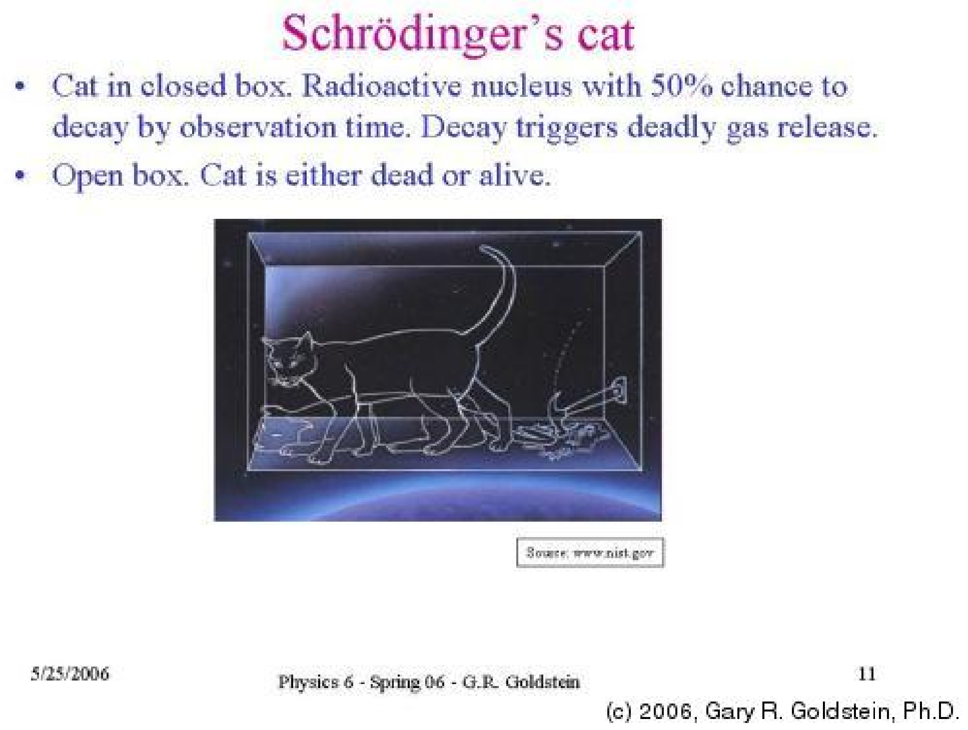
11. SchrÃ¶dingerÂ’s cat



Schrödinger's cat

- Cat in closed box. Radioactive nucleus with 50% chance to decay by observation time. Decay triggers deadly gas release.
- Open box. Cat is either dead or alive.

Source: www.nist.gov

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12. Schrödinger’s cat

Schrödinger's cat

- Cat in closed box. Radioactive nucleus with 50% chance to decay by observation time. Decay triggers deadly gas release.
- Open box. Cat is either dead or alive.
- Until observation cat is in mixed state - dead and alive!
- Is cat a particle? A system? What is Ψ for cat, dead or alive? Ψ for macroscopic system?
- Observation for mutually exclusive possibilities yields one outcome - collapse of the wavefunction

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