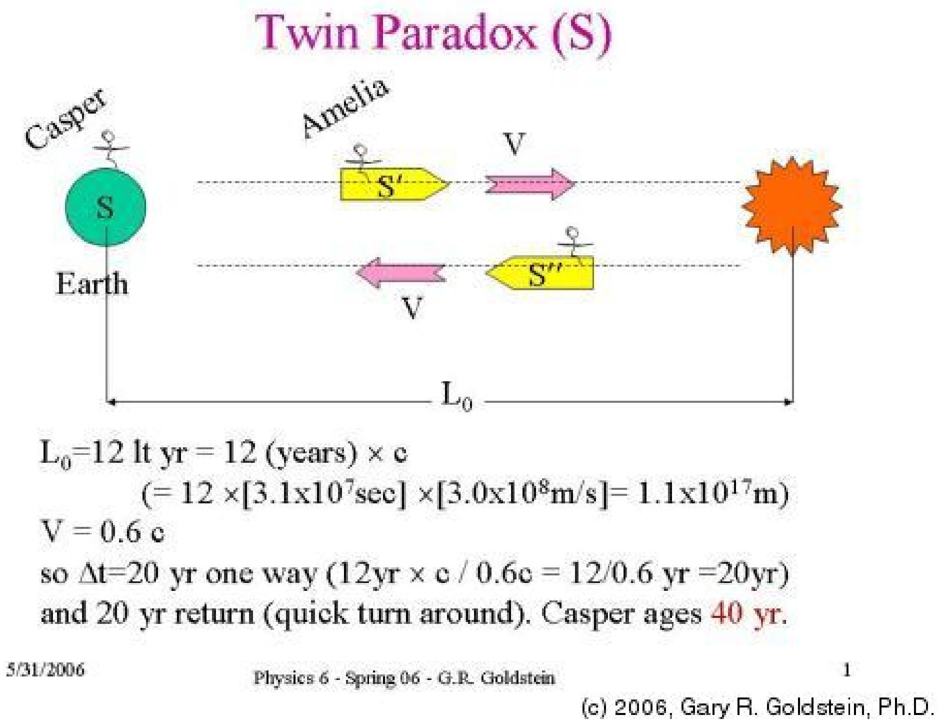
1. Twin Paradox (S)



Twin Paradox (S)

L_0=12 lt yr = 12 (years) × c
(= 12 ×[3.1×10^7sec] ×[3.0×10^8m/s]= 1.1×10^{17}m)
V = 0.6 c
so Δt=20 yr one way (12yr × c / 0.6c = 12/0.6 yr =20yr)
and 20 yr return (quick turn around). Casper ages 40 yr.

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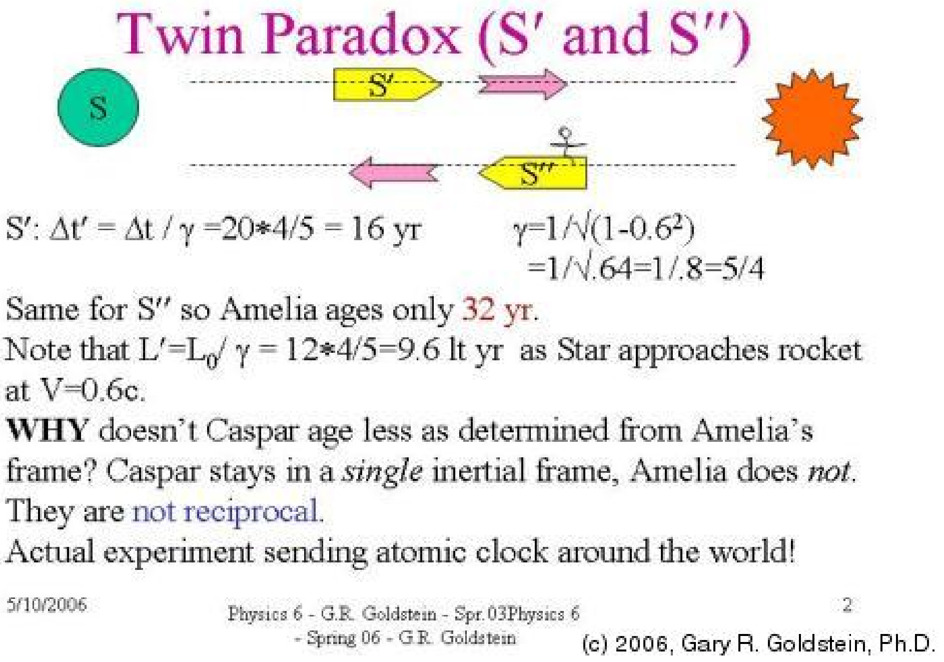
2. Twin Paradox (S? and S??)



Twin Paradox (S′ and S″)

S′: $\Delta t' = \Delta t / \gamma = 20*4/5 = 16$ yr $\gamma=1/\surd(1-0.6^2)$
$=1/\surd.64=1/.8=5/4$

Same for S″ so Amelia ages only 32 yr.
Note that $L'=L_0/\gamma = 12*4/5=9.6$ lt yr as Star approaches rocket at V=0.6c.
WHY doesn't Caspar age less as determined from Amelia's frame? Caspar stays in a *single* inertial frame, Amelia does *not*.
They are not reciprocal.
Actual experiment sending atomic clock around the world!

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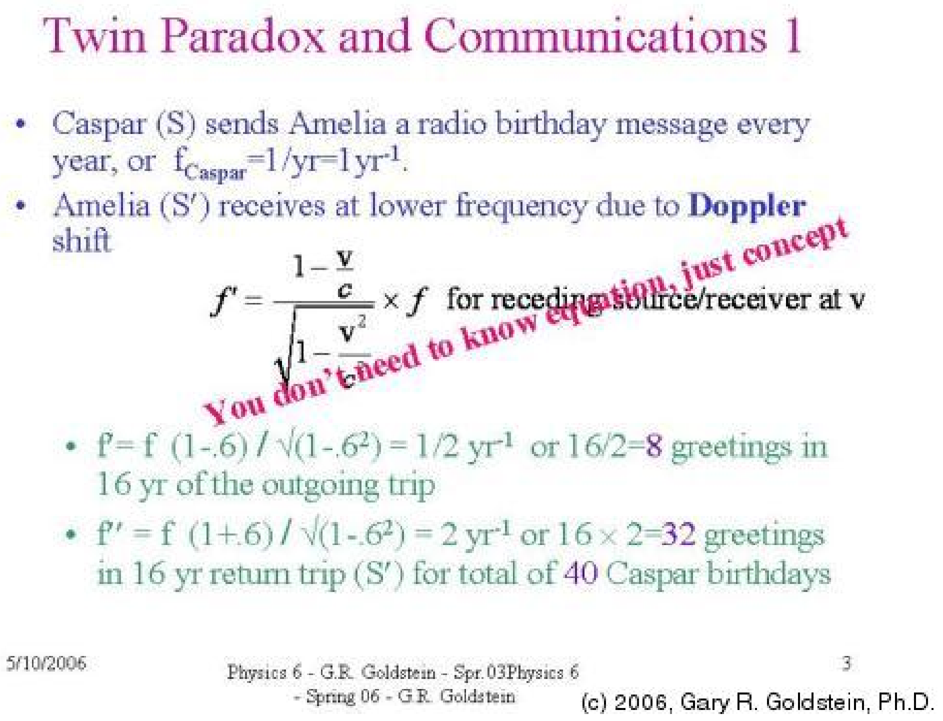
3. Twin Paradox and Communications 1



Twin Paradox and Communications 1

- Caspar (S) sends Amelia a radio birthday message every year, or $f_{Caspar}=1/yr=1yr^{-1}$.
- Amelia (S′) receives at lower frequency due to **Doppler** shift

$$f' = \frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \times f$$ for receding source/receiver at v

You don't need to know equation, just concept

  - $f' = f\ (1-.6) / \sqrt{(1-.6^2)} = 1/2\ yr^{-1}$ or 16/2=8 greetings in 16 yr of the outgoing trip
  - $f' = f\ (1+.6) / \sqrt{(1-.6^2)} = 2\ yr^{-1}$ or 16 × 2=32 greetings in 16 yr return trip (S′) for total of 40 Caspar birthdays

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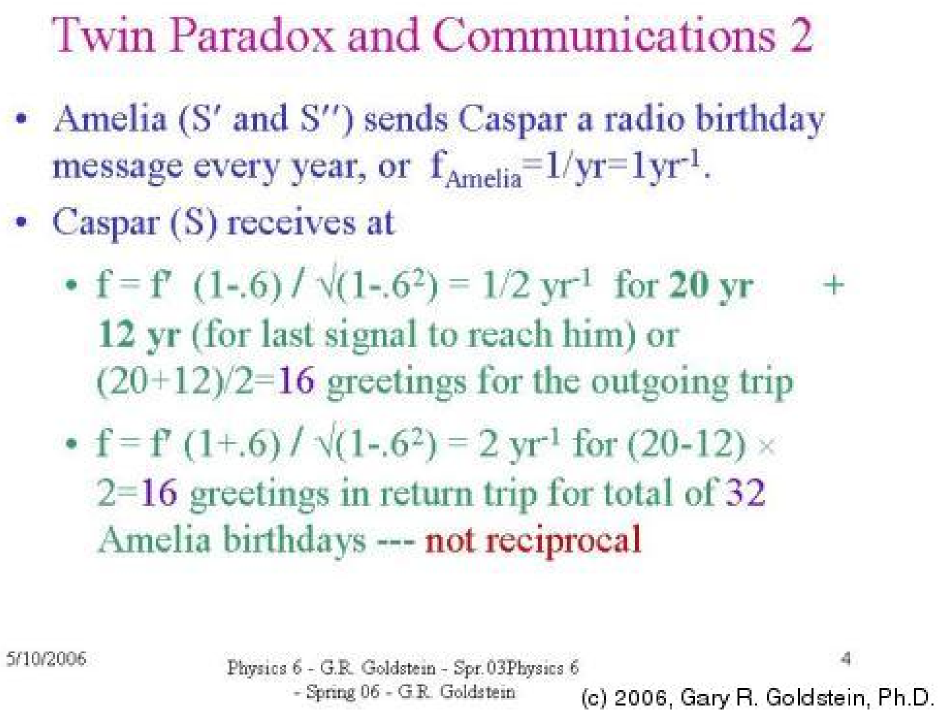
4. Twin Paradox and Communications 2



Twin Paradox and Communications 2

- Amelia (S′ and S″) sends Caspar a radio birthday message every year, or $f_{Amelia}=1/yr=1yr^{-1}$.
- Caspar (S) receives at
  - $f = f'\ (1-.6) / \sqrt{(1-.6^2)} = 1/2\ yr^{-1}$ for **20 yr** + **12 yr** (for last signal to reach him) or (20+12)/2=16 greetings for the outgoing trip
  - $f = f'\ (1+.6) / \sqrt{(1-.6^2)} = 2\ yr^{-1}$ for (20-12) × 2=16 greetings in return trip for total of 32 Amelia birthdays --- not reciprocal

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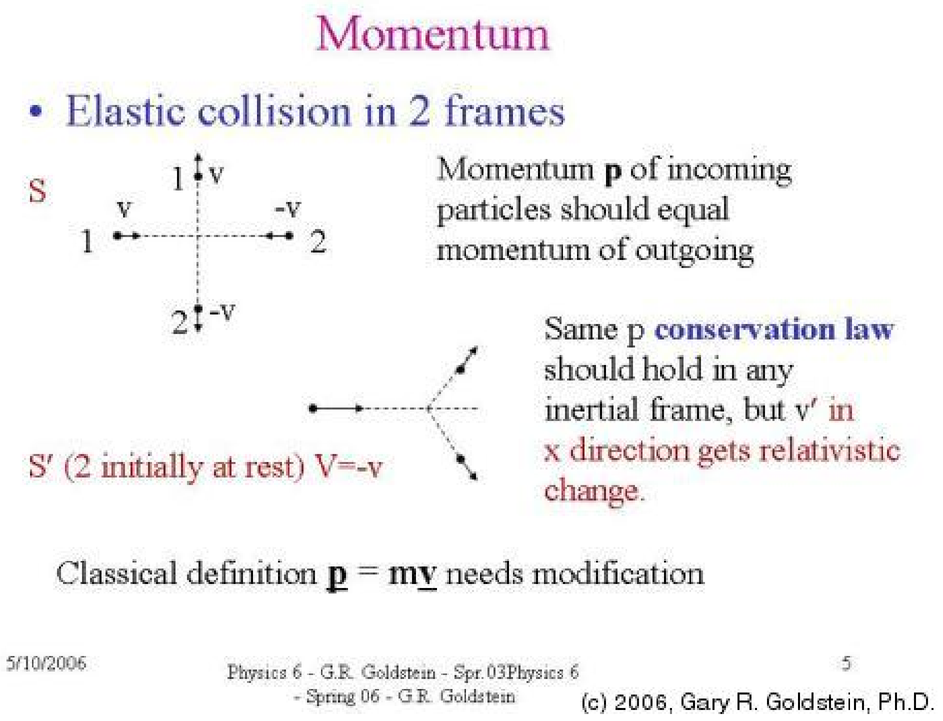
5. Momentum



Momentum

- Elastic collision in 2 frames

Momentum **p** of incoming particles should equal momentum of outgoing

Same p **conservation law** should hold in any inertial frame, but v′ in x direction gets relativistic change.

S′ (2 initially at rest) V=-v

Classical definition $\underline{\mathbf{p}} = \mathbf{m}\underline{\mathbf{v}}$ needs modification

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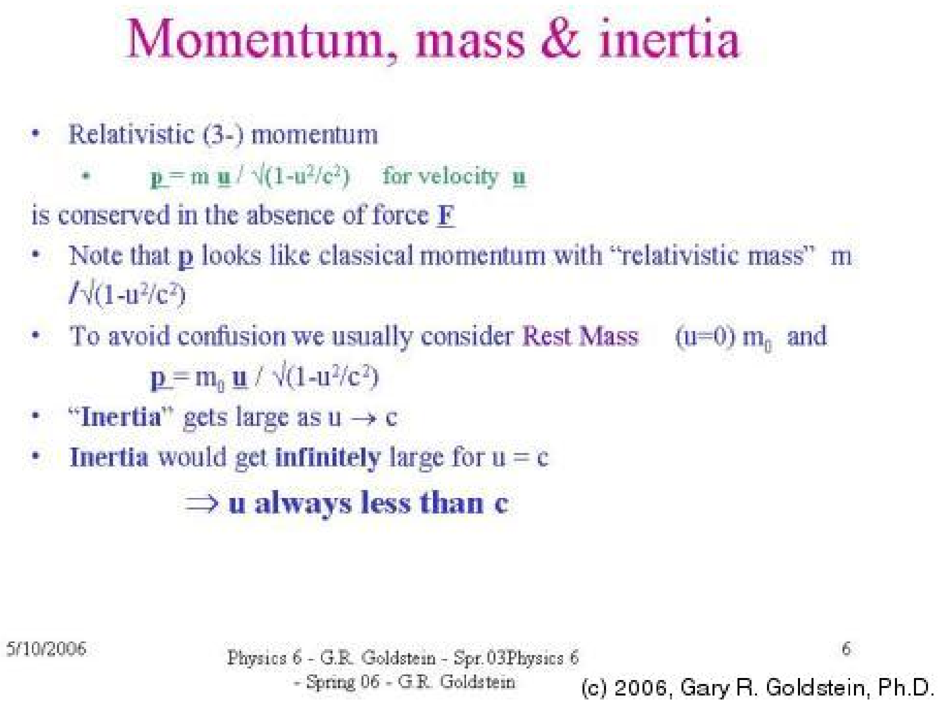
6. Momentum, mass & inertia



Momentum, mass & inertia

- Relativistic (3-) momentum
  - $\underline{p} = m\,\underline{u} / \sqrt{(1-u^2/c^2)}$ for velocity $\underline{u}$

is conserved in the absence of force $\underline{F}$

- Note that $\underline{p}$ looks like classical momentum with "relativistic mass" $m / \sqrt{(1-u^2/c^2)}$
- To avoid confusion we usually consider Rest Mass (u=0) m_0 and
  $\underline{p} = m_0\,\underline{u} / \sqrt{(1-u^2/c^2)}$
- "**Inertia**" gets large as u → c
- **Inertia** would get **infinitely** large for u = c

⇒ u always less than c

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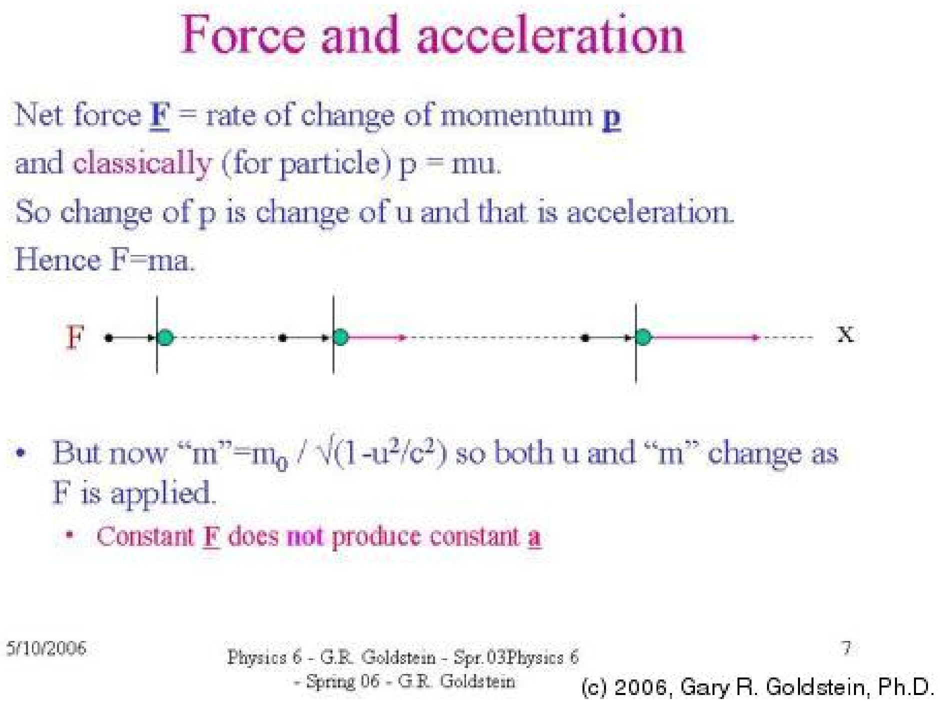
7. Force and acceleration



Force and acceleration

Net force **F** = rate of change of momentum **p**
and classically (for particle) $p = mu$.
So change of p is change of u and that is acceleration.
Hence $F=ma$.

- But now "m"$=m_0 / \sqrt{(1-u^2/c^2)}$ so both u and "m" change as F is applied.
  - Constant **F** does **not** produce constant **a**

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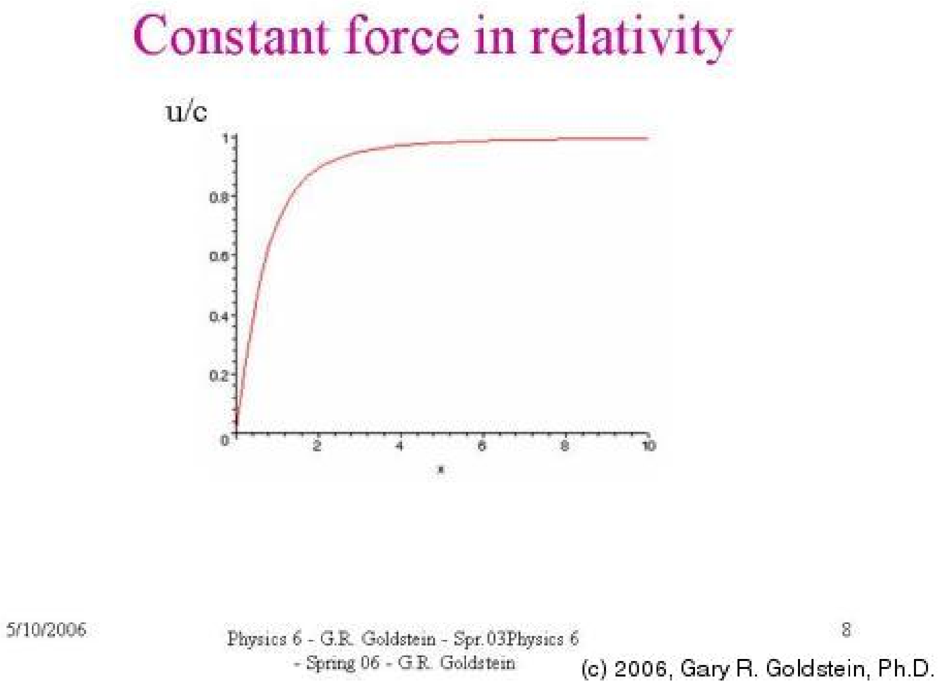
8. Constant force in relativity



Constant force in relativity

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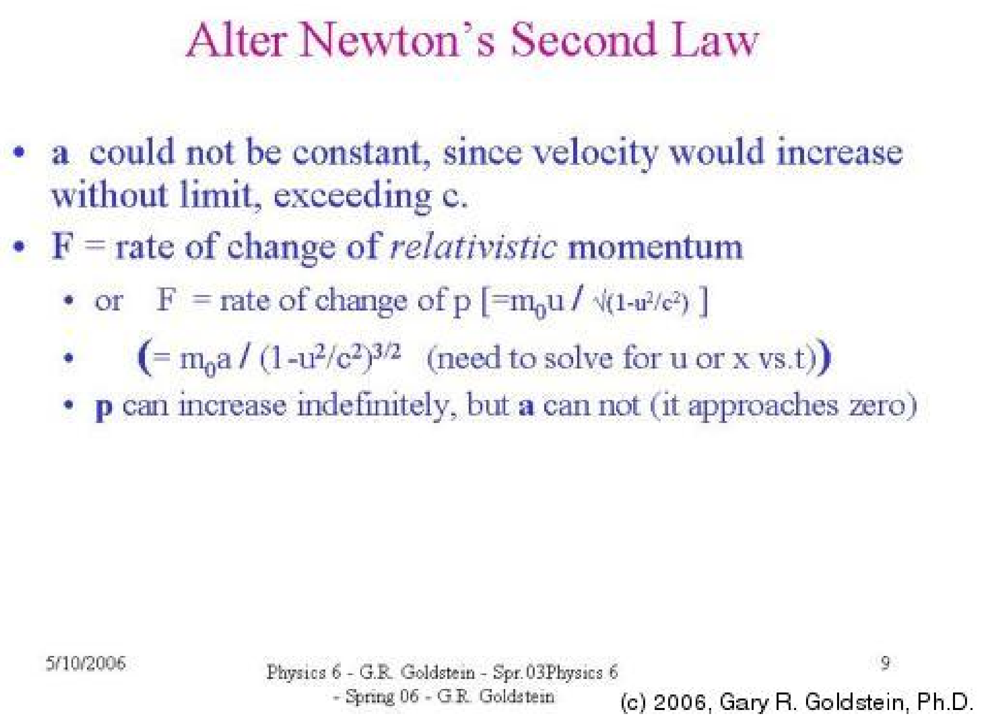
9. Alter Newton’s Second Law



Alter Newton’s Second Law

- **a** could not be constant, since velocity would increase without limit, exceeding c.
- F = rate of change of *relativistic* momentum
  - or F = rate of change of p [$=m_0u / \sqrt{(1-u^2/c^2)}$]
  - ($= m_0a / (1-u^2/c^2)^{3/2}$ (need to solve for u or x vs.t))
  - **p** can increase indefinitely, but **a** can not (it approaches zero)

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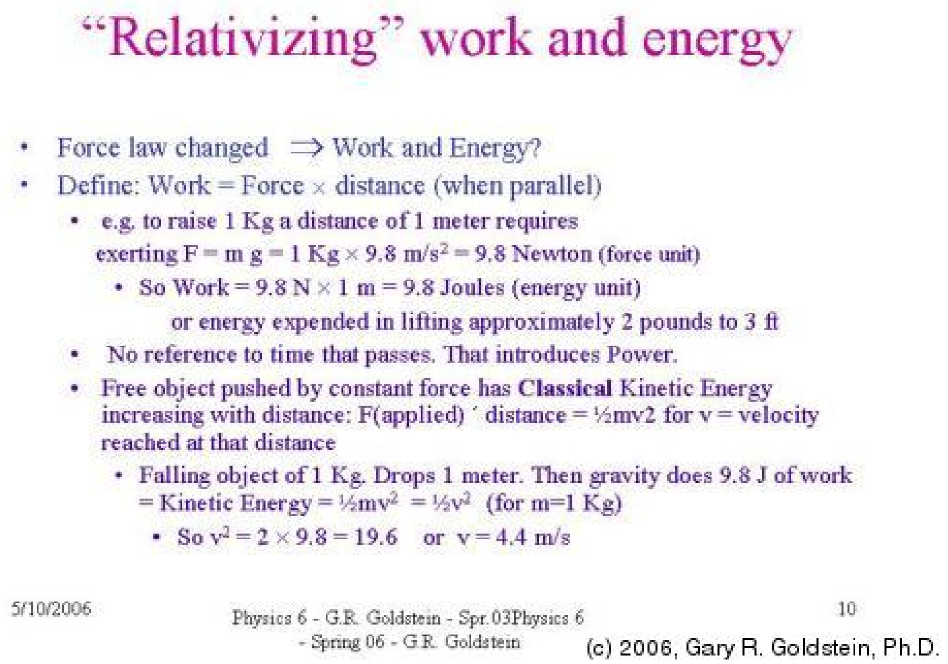
10. “Relativizing” work and energy



“Relativizing” work and energy

- Force law changed ⇒ Work and Energy?
- Define: Work = Force × distance (when parallel)
  - e.g. to raise 1 Kg a distance of 1 meter requires exerting F = m g = 1 Kg × 9.8 m/s^2 = 9.8 Newton (force unit)
    - So Work = 9.8 N × 1 m = 9.8 Joules (energy unit) or energy expended in lifting approximately 2 pounds to 3 ft
  - No reference to time that passes. That introduces Power.
  - Free object pushed by constant force has **Classical** Kinetic Energy increasing with distance: F(applied) ´ distance = ½mv2 for v = velocity reached at that distance
    - Falling object of 1 Kg. Drops 1 meter. Then gravity does 9.8 J of work = Kinetic Energy = $\frac{1}{2}mv^2 = \frac{1}{2}v^2$ (for m=1 Kg)
      - So $v^2 = 2 \times 9.8 = 19.6$ or $v = 4.4$ m/s

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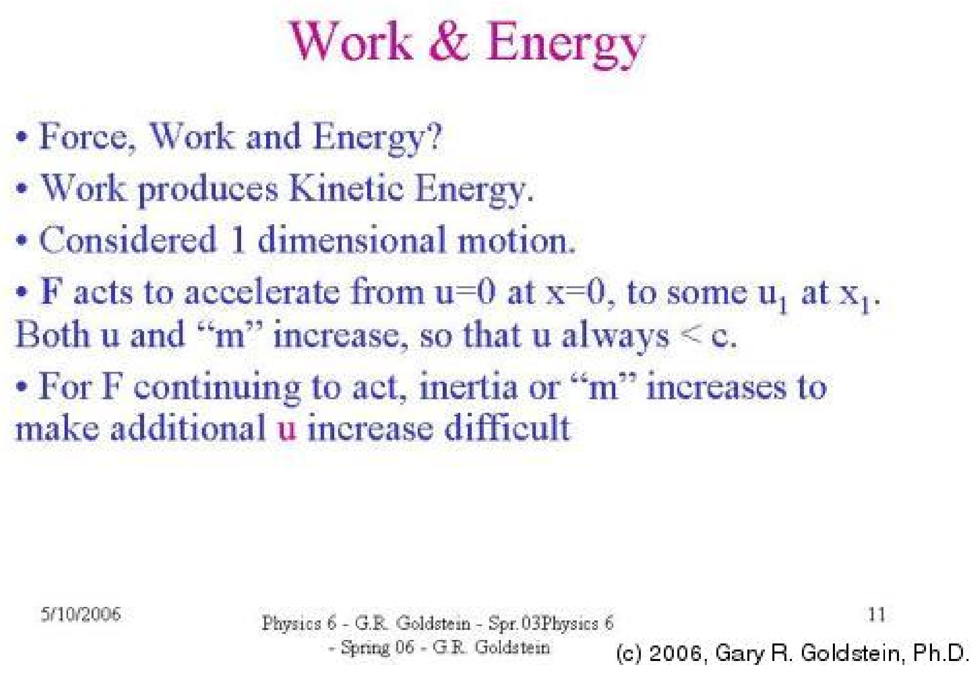
11. Work & Energy



Work & Energy

- Force, Work and Energy?
- Work produces Kinetic Energy.
- Considered 1 dimensional motion.
- F acts to accelerate from u=0 at x=0, to some u_1 at x_1. Both u and "m" increase, so that u always < c.
- For F continuing to act, inertia or "m" increases to make additional u increase difficult

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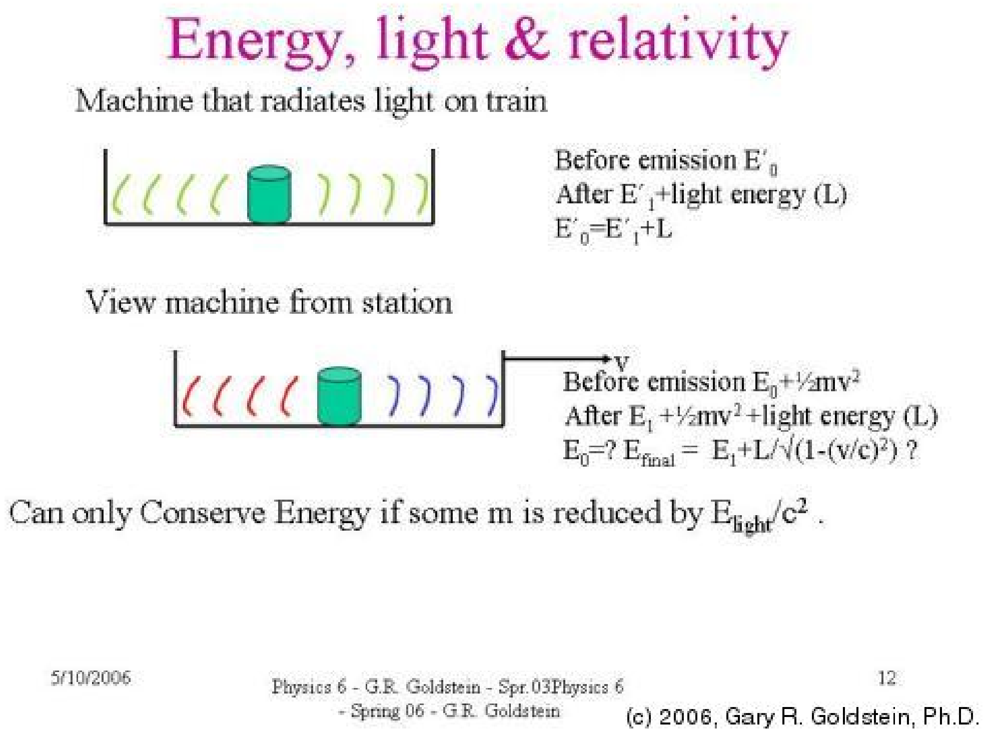
12. Energy, light & relativity



Energy, light & relativity

Machine that radiates light on train

Before emission E'_0
After E'_1+light energy (L)
$E'_0=E'_1+L$

View machine from station

Before emission $E_0+\frac{1}{2}mv^2$
After $E_1+\frac{1}{2}mv^2$ +light energy (L)
$E_0=?\ E_{final} = E_1+L/\sqrt{(1-(v/c)^2)}$?

Can only Conserve Energy if some m is reduced by E_{light}/c^2 .

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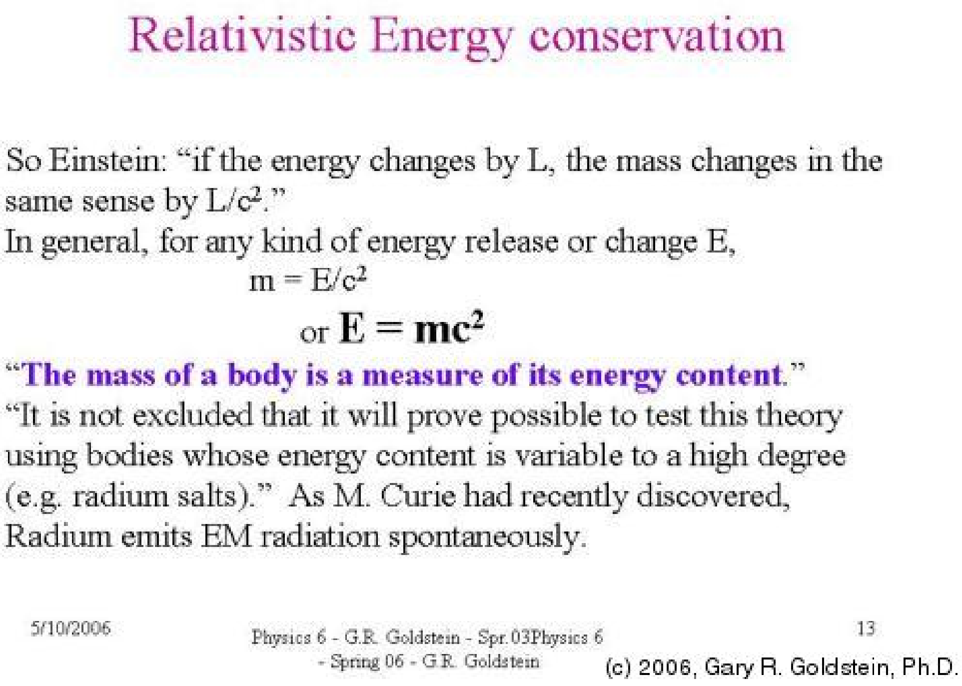
13. Relativistic Energy conservation



Relativistic Energy conservation

So Einstein: "if the energy changes by L, the mass changes in the same sense by L/c^2."
In general, for any kind of energy release or change E,

$$m = E/c^2$$

$$\text{or } \mathbf{E = mc^2}$$

"**The mass of a body is a measure of its energy content**."
"It is not excluded that it will prove possible to test this theory using bodies whose energy content is variable to a high degree (e.g. radium salts)." As M. Curie had recently discovered, Radium emits EM radiation spontaneously.

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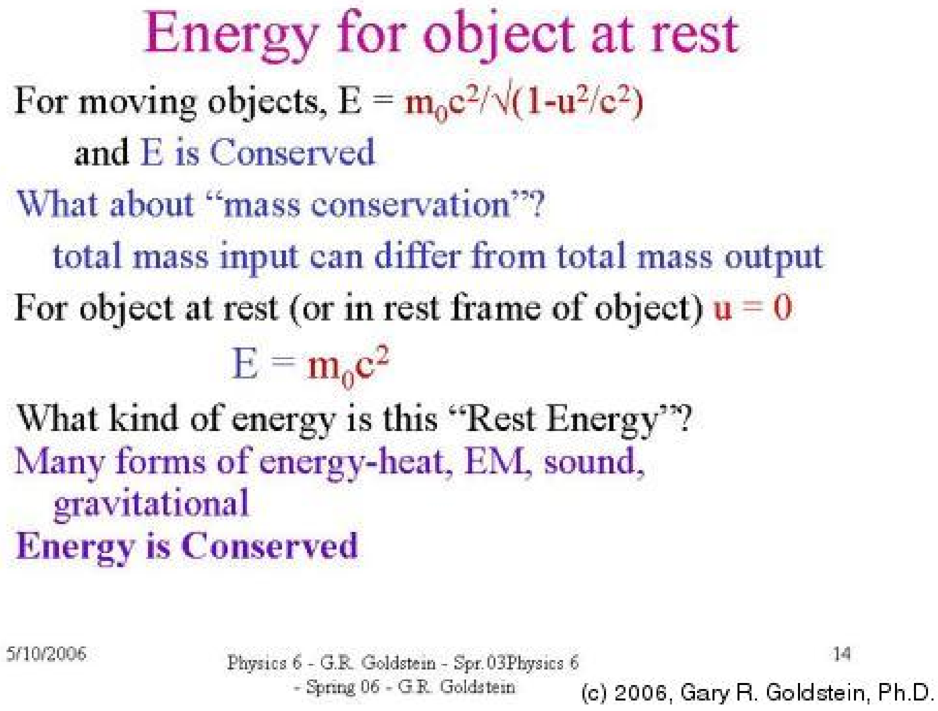
14. Energy for object at rest



Energy for object at rest

For moving objects, $E = m_0c^2/\sqrt{(1-u^2/c^2)}$
and E is Conserved
What about "mass conservation"?
total mass input can differ from total mass output
For object at rest (or in rest frame of object) $u = 0$

$$E = m_0c^2$$

What kind of energy is this "Rest Energy"?
Many forms of energy-heat, EM, sound, gravitational
Energy is Conserved

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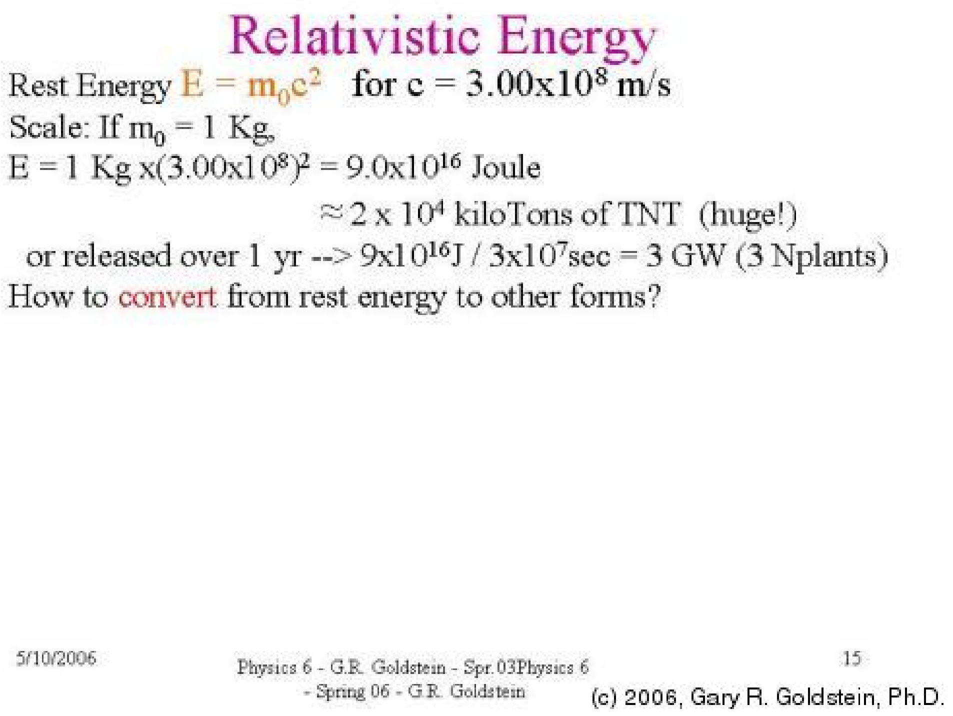
15. Relativistic Energy



Relativistic Energy

Rest Energy $E = m_0c^2$ **for $c = 3.00\text{x}10^8$ m/s**

Scale: If m_0 = 1 Kg,

$E = 1$ Kg $\text{x}(3.00\text{x}10^8)^2 = 9.0\text{x}10^{16}$ Joule

$\approx 2 \text{ x } 10^4$ kiloTons of TNT (huge!)

or released over 1 yr --> $9\text{x}10^{16}$J / $3\text{x}10^7$sec = 3 GW (3 Nplants)

How to convert from rest energy to other forms?

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