1. Announcement

Announcement

- Exams
  - March 2
  - April 6
  - Final May 5

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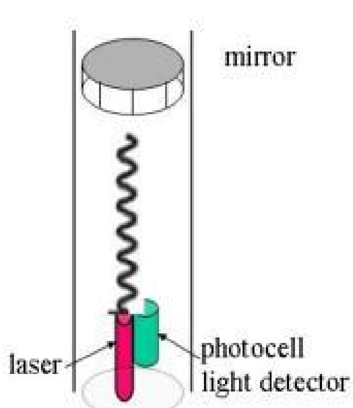
2. Construct light clock

Construct light clock



Light hits mirror & reflects back
To go 1 meter takes
$1/(3.0\times10^{8}\text{m/sec})$
$=3.3\times10^{-9}$ sec
=3.3 nanoseconds

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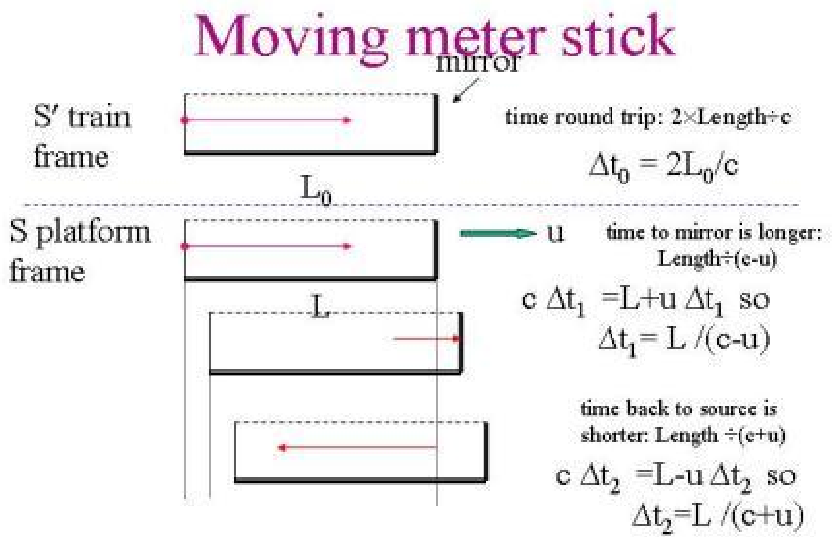
3. Moving meter stick



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4. Meter stick length

Meter stick length

- $\Delta t = \Delta t_1 + \Delta t_2 = 2L/[c(1-u^2/c^2)]$
- But $\Delta t = \Delta t_0/\sqrt{(1-u^2/c^2)}$ time dilation
- hence $2L/[c(1-u^2/c^2)] = 2L_0/[c\sqrt{(1-u^2/c^2)}]$
- or $L = L_0\sqrt{(1-u^2/c^2)} < L_0$ (proper length or length measured in "rest frame")
  - Length Contraction
  - moving meter stick "appears" short

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5. Lecture 6: Space-time, Velocity, Twins: Slide 5

Einstein's notes attempting to explain length contraction and time dilation to David Rothman without math!

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Sept.1939 - The above calculations are in Prof. Einstein's own hand, used to explain to me without the use of mathematics the reason for the contraction of a rod in the direction of its motion and why a clock changes its rhythm.
David A. Rothman

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6. Time and space are relative

Time and space are relative

- Duration (or time elapsed) of physical process depends on Frame
  - Shortest duration in Rest Frame of process ($\Delta t' = \Delta t_0$) "moving clock appears slow"
  - All processes are basically physical
- Size of phenomenon (or lengths) depends on Frame
  - Longest length (along motion) in Rest Frame of measuring stick ($\Delta L' = \Delta L_0$) "moving meter stick appears short"
- Primed frame is train (with fixed clock & meter stick) moving at v relative to station's unprimed frame

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta L = \Delta L' \sqrt{1 - \frac{v^2}{c^2}}$$

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7. Examples

Examples

- Clocks on nearby satellite vs. Earth observers
  - Circles earth in 90 minutes
    $=60\times90$ sec$=5400$ s $= 5.4\times10^3$s
  - Circumference = 24,000 mi = 39,000 Km
    $= 3.9\times10^7$m
  - Speed v= 3.9×10^7m / 5.4×10^3s = 7.2×10^3 m/s
  - Dilation factor $1/\sqrt{(1-v^2/c^2)}$
  - $v/c = 7.2\times10^3 / 3.0\times10^8 = 2.4\times10^{-5}$
  - $(v/c)^2=5.8\times10^{-10}$ or 0.58 per billion
  - then $\sqrt{(1-v^2/c^2)}$ less than 1 by 0.3 per billion &
  - $1/\sqrt{(1-v^2/c^2)}$ is 1 plus 0.3×10^{-9} or 0.3 nanoseconds longer than 1 sec on satellite's clocks

    (Need 11 place accuracy for small correction)
- Size of satellite vs. Earth observers - appears to be shorter by 0.3 parts per billion
  - If satellite is 30 m long it "appears" $30\times0.3\times10^{-9} = 9.0\times10^{-9}$ m shorter

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8. Keeping more accuracy

Keeping more accuracy

- > evalf(1/sqrt(1-((3.9*10^7/(5.4*10^3))^2/(3.0*10^8)^2)),24);
- 1.00000000030000000013500
- > evalf(%-1.0,24);
- $3.0000000013500*10^{-10}$
- > evalf(1/sqrt(1-((3.9*10^7/(5.4*10^3))^2/(3.0*10^8)^2)),12);
- 1.00000000030
- > evalf(%-1.0,12);
- $3.0*10^{-10}$

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9. Simultaneity is relative

Simultaneity is relative

Two *events* in S′ (e.g. train's frame):

2 lights (x_1'=0) and (x_2'=a) turned on simultaneously at t′=0

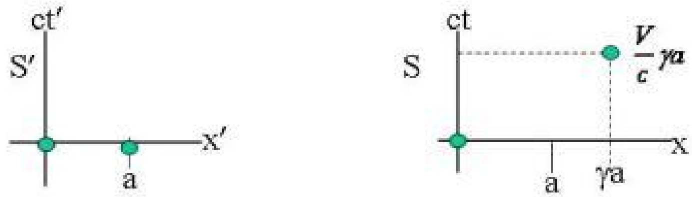
An *Event* is a space-time point.

Let S′ frame be moving with V relative to stationary S

In S: $x_1 = \gamma x_1' = 0$, $t_1 = 0$ (using $\gamma = 1/\sqrt{(1-V^2/c^2)}$)

$x_2 = \gamma a$, $t_2 = \gamma Va/c^2$ which is later ($\gamma > 1$)

2nd event appears later & further away

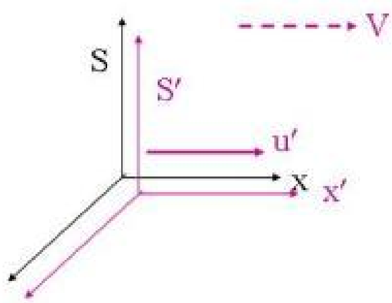


10. Velocity transformation

Velocity transformation

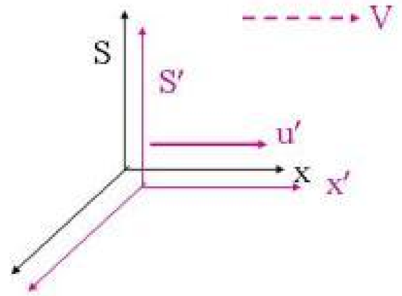
- In S′: object moving with velocity u̲′
  - S′ moves at V relative to S
- Classically get **u** = (u′ + V)
  - Would have c exceeded
- Relativity: **u** = (u′ + V)/(1 + Vu′/c²)
- Example - light:

  u′=c ⇒ u=c



11. Velocity transformation - example

Velocity transformation - example



Suppose S′ is frame of spaceship with V=0.8c. It launches a probe from its deck with u′=0.9c. What does Earth viewer see?

$u=(0.8c+0.9c)/(1+0.8\times0.9)=1.7c\ /1.72\ =0.99\ c$

- Can not exceed c, no matter how close u′ & V are to c.

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12. Velocity transformation - examples

Velocity transformation - examples

- 2 rockets approach earth from opposite directions along one axis with
- u_1= 0.8c and u_2= -0.8c from earth frame
- In rocket 1 frame $u_2'=(u_2-u_1)/(1+[-u_1]u_2/c^2)$ $= -1.6c/(1+.8^2)= -0.98c$
- Relative speed can never exceed 1c

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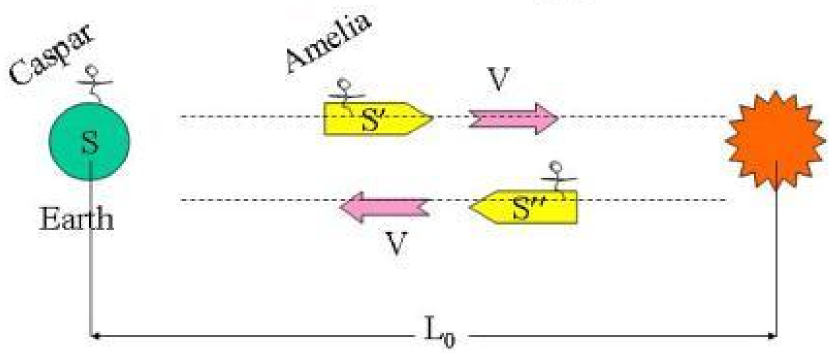
13. Twin Paradox (S)

Twin Paradox (S)



L_0=12 lt yr = 12 (years) × c
(= 12 ×[3.1x10^{7}sec] ×[3.0x10^{8}m/s]= 1.1x10^{17}m)
V = 0.6 c
so Δt=20 yr one way (12yr × c / 0.6c = 12/0.6 yr =20yr)
and 20 yr return (quick turn around). Casper ages 40 yr.

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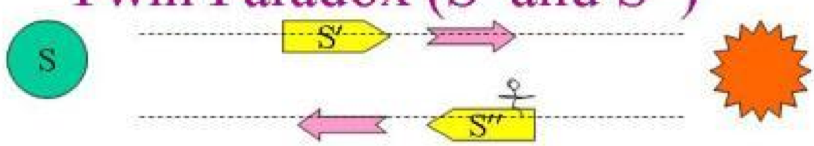
14. Twin Paradox (S' and S")

Twin Paradox (S′ and S″)



S′: $\Delta t' = \Delta t / \gamma$ =20∗4/5 = 16 yr $\gamma=1/\sqrt{(1-0.6^2)}$ $=1/\sqrt{.64}=1/.8=5/4$

Same for S″ so Amelia ages only 32 yr.
Note that $L'=L_0/\gamma$ = 12∗4/5=9.6 lt yr as Star approaches rocket at V=0.6c.
WHY doesn't Caspar age less as determined from Amelia's frame? Caspar stays in a *single* inertial frame, Amelia does *not*. They are not reciprocal.
Actual experiment sending atomic clock around the world!

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